Print Your Name Here: _____

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- ***Books, notes (electronic or paper), cell phones, smart phones, and internet-connected devices are prohibited!*** A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers, such as $\sqrt{2}$ or π, with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems and the *Maximum total score* = 100.

1. Use integration by parts to find $\int (\ln x)^2 \, dx$.

2. Use a trigonometric identity for $2 \sin x \cos x$ to find $\int_0^{\frac{\pi}{8}} \sin^2 x \cos^2 x \, dx$.

3. Use a trigonometric substitution to find $\int \frac{1}{(1+x^2)^{\frac{3}{2}}}\,dx$.

4. Use a partial fractions decomposition to find $\int \frac{x+1}{(x+2)(x-2)}\,dx$.

5. Evaluate the improper integral $\int_0^\infty xe^{-x^2}\,dx$ and state whether the integral converges or diverges.

Solutions

1. $\displaystyle\int (\ln x)^2\,dx = x(\ln x)^2 - \int 2\ln x\,dx = x(\ln x)^2 - \left[2x\ln x - \int 2\,dx\right] = x(\ln x)^2 - 2x\ln x + 2x + C.$
The most common errors were with the chain rule or else not realizing that udv must equal $(\ln x)^2\,dx$.

2. We note first that $\sin 2x = 2\sin x\cos x$, so that
$\displaystyle\int_0^{\frac{\pi}{8}} \sin^2 x\cos^2 x\,dx = \int_0^{\frac{\pi}{8}} \frac{1}{4}\sin^2 2x\,dx = \int_0^{\frac{\pi}{8}} \frac{1}{8}(1-\cos 4x)\,dx = \frac{\pi-2}{64}$. It would also be possible to solve this problem correctly by expressing both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$.

3. Draw a right triangle for which $\theta = \tan^{-1} x$. Then $\displaystyle\int \frac{1}{(1+x^2)^{\frac{3}{2}}}\,dx = \int \cos\theta\,d\theta = \frac{x}{\sqrt{1+x^2}} + C.$

4. $\displaystyle\int \frac{x+1}{(x+2)(x-2)}\,dx = \int \frac{\frac{1}{4}}{x+2} + \frac{\frac{3}{4}}{x-2}\,dx = \frac{1}{4}\ln|x+2| + \frac{3}{4}\ln|x-2| + C.$

5. $\displaystyle\int_0^\infty xe^{-x^2}\,dx = \frac{1}{2}\int_0^\infty e^{-u}\,du = \lim_{b\to\infty}\frac{1}{2}\int_0^b e^{-u}\,du = \frac{1}{2}$, so the integral converges.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test 4	Final Exam	Final Grade
90-100 (A)	8					
80-89 (B)	7					
70-79 (C)	7					
60-69 (D)	4					
0-59 (F)	11					
Test Avg	72.2%	%	%	%	%	%