Print Your Name Here: ____________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- ***Books, notes (electronic or paper), cell phones, smart phones, and internet-connected devices are prohibited!*** A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers, such as $\sqrt{2}$, π, or $\sin\frac{\pi}{7}$ with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems of equal weight and the *Maximum total score* = 100.

1. Find an *equation* for the *straight line tangent* to the graph of the parametric equations $x = 2\cos t,\ y = \sin t$ at the point (x, y) corresponding to $t = \dfrac{\pi}{4}$.

2. For the graph of the polar equation $r = 2\sin\theta$ find the *slope of the tangent line* at the point corresponding to $\theta = \dfrac{\pi}{6}$. Also, write a *rectangular equation* equivalent to the given polar equation, and identify the *center and radius* of the graph, which is a circle.

3. Find the area enclosed by *one loop* of the graph of the polar equation $r = \sin 3\theta$. (Suggestion: It may help to sketch one loop in order to obtain the correct upper and lower limits of integration.)

4. Find the length of the arc given in polar coordinates by $r = \theta^2$, $0 \leq \theta \leq 2$.

5. The polar equation $r = \dfrac{3}{3 - 2\cos\theta}$ represents a conic section. Find:

a. The eccentricity e and the name of the type of conic section.

b. An equation using rectangular coordinates for the directrix line, using the focus for the origin.

c. The polar coordinates of the points corresponding to $\theta = 0$, $\frac{\pi}{2}$, π, $\frac{3}{2}\pi$. (Suggestion: It may help you to make a sketch although the drawing won't be graded.)

Solutions

1. $2y + x = 2\sqrt{2}$ or an equivalent. There are four main steps to solving this problem. First find $\dfrac{dy}{dx}$, then the slope of the tangent for the given value of t, and finally the rectangular coordinates of the point of tangency and an equation for the tangent line.

2. The value of the derivative, $\dfrac{dy}{dx} = \tan 2\theta$, at $\theta = \frac{\pi}{6}$ is $\sqrt{3}$. A serious common error was to confuse $\dfrac{dr}{d\theta}$ with $\dfrac{dy}{dx} = \tan 2\theta$ even though by chance both agree at the specified value of θ in this particular exercise. Multiplying both sides of the polar equation by r yields, $x^2 + y^2 = 2y$, or $x^2 + (y-1)^2 = 1$. This rectangular equation shows that the circle has center $(0, 1)$ and radius 1.

3. $A = \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{3}} \sin^2 3\theta \, d\theta = \dfrac{\pi}{12}$. Note that $\sin 3\theta$ grows from 0 to 1 as θ increases from 0 to $\frac{\pi}{6}$. And don't stop practicing your techniques of integration!

4. $L = \displaystyle\int_0^2 \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta = \int_0^2 \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^2 \theta\sqrt{\theta^2 + 4}\, d\theta = \frac{8}{3}\left(2\sqrt{2} - 1\right)$. We used a simple algebraic substitution to evaluate the latter integral, as in the homework. The following errors must be avoided: $\sqrt{a^2 + b^2} \neq a + b$ and $\sqrt{ab} \neq a + b$. The use of nonsense to avoid techniques of integration is unacceptable. If you do not know how to complete a solution, do not try to fake it.

5.

a. $e = \frac{2}{3}$ and the curve is an ellipse.

b. $x = -\frac{3}{2}$.

c. $(3, 0)$, $(1, \frac{\pi}{2})$, $(\frac{3}{5}, \pi)$, $(1, \frac{3}{2}\pi)$.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test #4	Final Exam	Final Grade
90-100 (A)	8	8				
80-89 (B)	7	5				
70-79 (C)	8	9				
60-69 (D)	4	2				
0-59 (F)	12	11				
Test Avg	71.8%	71.4%	%	%	%	%