Print Your Name Here: ______________________________

- Grading is based mainly on the **detailed work, which you must show** in the space provided—not just the answers. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- ***Books, notes (electronic or paper), cell phones, smart phones, and internet-connected devices are prohibited!*** A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do not replace precise answers, such as $\sqrt{2}$, π, or $\sin\frac{\pi}{7}$ with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems of equal weight and the *Maximum total score* = 100.

1. *Find all values* of x for which the *geometric* series $\displaystyle\sum_{n=0}^{\infty}\left(\frac{x-1}{2}\right)^n$ converges. *To what function* $f(x)$ does it converge?

2. Use the *integral test* to determine whether or not the series $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ converges.

3. Use a *limit comparison* test to determine whether $\displaystyle\sum_{n=0}^{\infty}\frac{n+1}{\sqrt{n^5+1}}$ converges or diverges.

4. Test for absolute convergence, conditional convergence, or divergence: $\sum_{n=0}^{\infty}(-1)^n \frac{n}{2^n}$.

5. Let $f(x) = e^x$.

a. (10) *Use Taylor's coefficient formula* to expand $f(x)$ in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$.

b. (5) Use the result of part (a) to express e^{-x^2} as the sum of a power series.

c. (5) Use the result of part (b) to express $\int e^{-x^2}\, dx$ as the sum of a power series.

Solutions

1. For $-1 < x < 3$ the series converges to $f(x) = \dfrac{2}{3-x}$, which we can see using the formula for the sum of a geometric series. I counted 15 points for the interval of convergence and 5 for the sum.

2. The series diverges because $\displaystyle\int_2^\infty \frac{1}{x \ln x}\, dx = \infty$.

3. This series converges by means of a limit comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$, a convergent p-series. The work of the limit comparison must be shown.

4. The series is absolutely convergent by the ratio test.

5.

a. $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

b. $e^{-x^2} = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$, replacing x in part (a) with $-x^2$.

c. $\displaystyle\int e^{-x^2}\, dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)n!}$

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test #4	Final Exam	Final Grade
90-100 (A)	8	10	5			
80-89 (B)	7	4	2			
70-79 (C)	8	9	9			
60-69 (D)	4	3	11			
0-59 (F)	9	10	6			
Test Avg	73.5%	71.4%	70.4%	%	%	%