Print Your Name Here: _____

- Grading is based mainly on the **detailed work, which you must show** in the space provided—not just the answers. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- ***Books, notes (electronic or paper), communication devices (eg laptops, tablets, cell/smart phones, I-watches) are prohibited!*** A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed but not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do *not* replace precise answers, such as $\sqrt{2}$, π, or $\sin\frac{\pi}{7}$ with decimal approximations. Keep your eyes on your own paper!
- There are **five (5)** problems and the *Maximum total score* = 100.

1. Let $\mathbf{a} = \langle 2, 1, 2\rangle$, $\mathbf{b} = \langle 3, 2, 1\rangle$ and let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$, $0 \le \theta \le \pi$.

a. (15) Find $|\mathbf{a}|$, $|\mathbf{b}|$ and $\cos\theta$.

b. (10) Find $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$. (These two are known also as the *scalar* projection and the *vector* projection of $\mathbf{b}$ onto $\mathbf{a}$.)

c. (5) Find a unit vector $\mathbf{u}$ having the same direction as $\mathbf{a}$, and identify the three *direction cosines* of $\mathbf{a}$.

2. (10) Find the point(s) of intersection of the curve $\mathbf{r}(t) = \langle t, t+1, 1-t^2\rangle$ with the surface $z = y^2 - x^2$.

3. (20) Let $\mathbf{a} = \langle 2, 1, 2\rangle$ and $\mathbf{b} = \langle 1, 1, 2\rangle$. Find $\mathbf{a} \times \mathbf{b}$ and also find the volume of the parallelepiped spanned by the vectors $\mathbf{a}$, $\mathbf{b}$ and a third vector $\mathbf{c} = \langle 1, 1, 1\rangle$, which form the three edges emanating from one vertex.

4. (20) Let P be the point $(1, 2, 3)$ and $\mathbf{n} = \langle -1, 3, -2\rangle$. Find

a. an equation for the *plane* through P with *normal* vector $\mathbf{n}$.

b. a vector equation for the straight line through P with direction given by $\mathbf{n}$.

5. Let $\mathbf{r}(t) = \langle \cos 2t, \sin 2t, t\rangle$. Find:

a. (12) $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

b. (8) $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ and the angle between these two vectors.

Solutions

1. Be careful not to confuse vectors with scalars. A scalar is *never* equal to a vector.

a. $|\mathbf{a}| = 3,\ |\mathbf{b}| = \sqrt{14}$ and $\cos\theta = \frac{10}{3\sqrt{14}}$.

b. $\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{10}{3}$ and $\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \frac{10}{9}\langle 2,1,2\rangle$.

c. $\mathbf{u} = \frac{1}{3}\langle 2,1,2\rangle$ and the three direction cosines are $\frac{2}{3},\ \frac{1}{3},\ \frac{2}{3}$.

2. $(0,1,1)$ and $(-2,-1,-3)$. Most errors were with algebra.

3. $\mathbf{a}\times\mathbf{b} = \langle 0,-2,1\rangle$ and the volume is $|\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})| = 1$. The most common error was omission of the absolute value from the volume formula. Volumes are *positive*.

4. An equation *must* have an equal sign. An equation of a plane is satisfied by (x,y,z) if and only if (x,y,z) lies on the designated plane.

a. $x - 3y + 2z = 1$ or an equivalent equation.

b. $\mathbf{r}(t) = \langle 1,2,3\rangle + t\langle -1,3,-2\rangle$ or an equivalent equation.

5.

a. $\mathbf{r}'(t) = \langle -2\sin 2t, 2\cos 2t, 1\rangle$ and $\mathbf{r}''(t) = \langle -4\cos 2t, -4\sin 2t, 0\rangle$.

b. $\mathbf{r}'(t)\cdot\mathbf{r}''(t) = 0$, so the angle between these two vectors is $\frac{\pi}{2}$, a right angle. The vectors had to be perpendicular since $|\mathbf{r}'(t)|$ is a constant.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test #4	Final Exam	Final Grade
90-100 (A)	8	10	5	16		
80-89 (B)	7	4	2	7		
70-79 (C)	8	9	9	5		
60-69 (D)	4	3	11	2		
0-59 (F)	9	10	8	3		
Test Avg	73.5%	71.4%	70.3%	84.1%	%	%