Print Your Name Here: ________________________________

- Grading is based mainly on the **detailed work, which you must show** in the space provided—not just the answers. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side*, and write your name at the top of the scratch sheet if you will turn it in for grading.
- ***Books, notes (electronic or paper), communication devices (eg laptops, tablets, cell/smart phones, I-watches) are prohibited!*** A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed but not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. Do *not* replace precise answers, such as $\sqrt{2}$, π, or $\sin\frac{\pi}{7}$ with decimal approximations. Keep your eyes on your own paper!
- There are **ten (10)** problems of equal weight and the *Maximum total score* = 200.

1. Use integration by parts to find $\int x\sin\pi x\,dx$.

2. Use a trigonometric substitution to find $\int_0^2 x^2\sqrt{4-x^2}\,dx$.

3. Evaluate the improper integral $\int_0^\infty \frac{x}{x^4+1}\,dx$.

4. At what point(s) on the parametric curve $x = 3t^2 + 2,\ y = t^3 - 3$ does the tangent line have slope $\dfrac{dy}{dx} = \dfrac{1}{2}$?

5. Find the *area* enclosed by the *cardioid*, given in polar coordinates by $r = 1 + \sin\theta$, $0 \le \theta \le 2\pi$.

6. Use the ratio test to find the *radius* R of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$. Also find the *interval* of convergence, being sure to test the two endpoints.

7. Use the Taylor coefficient formula to *find* the Maclaurin series (i.e. power series in powers of x) for $f(x) = \sin x$. (Suggestion: *find* a_0, a_1, a_2, a_3, notice the pattern, and express $f(x)$ as the sum of an infinite series of coefficients times powers of x.)

8. Let θ be the angle between the vectors $\mathbf{a} = \langle 3, 2, 1 \rangle$ and $\mathbf{b} = \langle 2, 3, 1 \rangle$. Find:

a. $\mathbf{a} \cdot \mathbf{b}$

b. $\cos\theta$

c. $\mathbf{a} \times \mathbf{b}$

d. the area of the parallelogram with adjacent edges **a** and **b**.

9. Consider two points $P(1,2,3)$ and $Q(3,1,2)$.

a. Find a vector equation for the straight line through P and Q.

b. Find an equation for the plane through the point P and perpendicular to the vector $\overrightarrow{PQ}$.

10. The position vector of a particle at time t is $\mathbf{r}(t) = \langle t, \cos t, \sin t \rangle$. Find:

a. The velocity vector $\mathbf{r}'(t)$.

b. The arc length traveled by $\mathbf{r}(t)$ for $0 \leq t \leq 2\pi$.

c. The unit tangent $\mathbf{T} = \dfrac{d\mathbf{r}}{ds}$.

d. $\dfrac{d\mathbf{T}}{ds}$, and the curvature κ at time t.

Solutions

1. $\int x \sin \pi x \, dx = \frac{\sin \pi x}{\pi^2} - \frac{x \cos \pi x}{\pi} + C$. Remember that udv needs to be the whole integrand when using parts.

2. Letting $x = 2\sin\theta$ we find that $\int_0^2 x^2\sqrt{4-x^2}\, dx = \pi$. It helps to know $\sin 2\theta$ and $\cos 2\theta$.

3. Substituting $u = x^2$ we obtain an integrand recognized as the derivative of the inverse tangent. Thus $\int_0^\infty \frac{x}{x^4+1}\, dx = \frac{1}{2}\int_0^\infty \frac{1}{u^2+1}\, du = \frac{1}{2}\left(\lim_{b\to\infty} \tan^{-1} b - \tan^{-1} 0\right) = \frac{\pi}{4}$. It is also possible to solve this problem with a trigonometric substitution $x^2 = \tan\theta$.

4. Since $\frac{dy}{dx} = \frac{t}{2} = \frac{1}{2}$ we find that $t = 1$ and the corresponding point on the curve is $(5, -2)$.

5. $A = \frac{1}{2}\int_0^{2\pi} r(\theta)^2\, d\theta = \frac{3}{2}\pi$. As we saw very often in class going over the homework problems, one really needs to know $\cos 2\theta$ to evaluate the integral of $\sin^2\theta$.

6. $R = 1$ by the ratio test, which one must know how to use, and the interval of convergence is $(-1, 1]$, where we have convergence when $x = 1$ by the alternating series test, and divergence when $x = -1$ since this results in minus the harmonic series, which diverges.

7. $a_0 = 0,\ a_1 = 1,\ a_2 = 0,\ a_3 = \frac{1}{3!}$, and $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ or an equivalent expression.

8.

a. $\mathbf{a}\cdot\mathbf{b} = 13$

b. $\cos\theta = \frac{13}{14}$

c. $\mathbf{a}\times\mathbf{b} = \langle -1, -1, 5\rangle$

d. the area of the parallelogram with adjacent edges $\mathbf{a}$ and $\mathbf{b}$ is $|\mathbf{a}\times\mathbf{b}| = |\langle -1, -1, 5\rangle| = 3\sqrt{3}$.

9.

a. $\mathbf{r}(t) = \langle 1+2t, 2-t, 3-t\rangle$ or an equivalent vector equation.

b. $2x - y - z + 3 = 0$ or an equivalent linear equation in 3 variables using the normal vector $\overrightarrow{PQ} = \langle 2, -1, -1\rangle$.

10.

a. $\mathbf{r}'(t) = \langle 1, -\sin t, \cos t\rangle$.

b. $\int_0^{2\pi} |\mathbf{r}'(t)|\, dt = 2\pi\sqrt{2}$ since $|\mathbf{r}'(t)| = \sqrt{2}$.

c. $\mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{1}{\sqrt{2}}\langle 1, -\sin t, \cos t\rangle$.

d. $\frac{d\mathbf{T}}{ds} = \frac{1}{2}\langle 0, -\cos t, -\sin t\rangle$, and the curvature $\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{1}{2}$.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Test #4	Final Exam	Final Grade
90-100 (A)	8	10	5	16	4	4
80-89 (B)	7	4	2	7	7	12
70-79 (C)	8	9	9	5	11	11
60-69 (D)	4	3	11	2	8	5
0-59 (F)	9	10	8	3	4	4
Test Avg	73.5%	71.4%	70.3%	84.1%	74.45%	74.31%