I have read, understood, and complied with the instructions in the box below. Legible

***Signature* and LSU ID #:** ______________________________

- Download and print a copy of this test on 8.5 by 11 inch letter size paper. If no printer is available, make a hand-written facsimile. Be sure to sign the statement above.
- **Show *All* Work** in the space provided. Grading is based on the correctness of the work shown to justify the answers. We can give credit *only* for what you write! *Indicate clearly if you continue a problem on a second page.* There are 5 problems worth 20 points each.
- ***You may use your text book, Zoom recordings of our class meetings, your class notes, and your homework!*** *However, no other sources or communication devices may be used.* ***All work must be your own.*** If you use a calculator, you *must still write out all operations performed* on the calculator. *Do not replace* precise answers, such as $\sqrt{2}$, π, or $\cos\frac{\pi}{7}$ with decimal approximations. *Make all obvious simplifications.* Submit only your own work!
- This is a take-home test on an *honor system.* You may take as much time as you like, but **I must receive your completed test by email no later than 12:00 noon on Saturday, January 30.** If you have no device that scans your work directly to a single pdf file, then photograph your pages in the correct order with your phone and save as jpeg, then try this please: put the jpeg files into your computer, highlight the whole group of pictures, right click PRINT and then select PRINT TO PDF. That way I can receive a multipage PDF file which is possible to grade in a way you will be able to read later. Email that file to me **rich@math.lsu.edu** as soon as you are ready but no later than Saturday, January 30, at 12:30 noon. *These instructions express my trust and confidence in your integrity and good character.*

Before you send me your pdf file containing all your pages as one single file, please make sure everything is legible. Use a sufficiently dark writing instrument for your test and make sharp, clear images, so I can read them. I simply cannot grade what I cannot read. Thank you for your consideration in this!

1. Use *integration by parts* to evaluate $\int 2x\tan^{-1}x\,dx$. (Hint: Let $u = \tan^{-1}x$. What then must be dv? Show how you check your answer.)

2. Evaluate $\int_0^{\frac{\pi}{8}} \sin^2 x \cos^2 x \, dx$. (Hint: Do you know an identity for $\sin 2x$?)

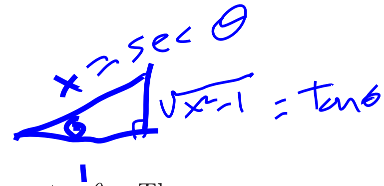
3. Use a *trigonometric substitution* to evaluate $\int \frac{\sqrt{x^2-1}}{x^2}\,dx$.

4. Use the *Partial Fractions* theorem to evaluate $\displaystyle\int \frac{3x^2+1}{(x-1)(x^2+1)}\,dx$.

5. Evaluate the improper integral $\displaystyle\int_3^\infty \frac{1}{x^2 - x}\,dx$ or else show that it does not converge if that is the case. (Hint: Use a partial fractions decomposition and the definition of the improper integral.)

Solutions

1. We need to let $dv = 2x\,dx$. Thus
$\int 2x\tan^{-1}x\,dx = x^2\tan^{-1}x - \int \frac{x^2}{1+x^2}\,dx = x^2\tan^{-1}x - \int 1 - \frac{1}{1+x^2}\,dx = (x^2+1)\tan^{-1}x - x + C$.
The answer should be checked by differentiation.



2. $\int_0^{\frac{\pi}{8}} \sin^2 x\cos^2 x\,dx = \int_0^{\frac{\pi}{8}} \frac{\sin^2 2x}{4}\,dx = \frac{1}{4}\int_0^{\frac{\pi}{8}} \frac{1-\cos 4x}{2}\,dx = \frac{\pi - 2}{64}$.

3. Using a right triangle, substitute $x = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$, and $\sqrt{x^2-1} = \tan\theta$. Then $\int \frac{\sqrt{x^2-1}}{x^2}\,dx = \int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \sec\theta - \cos\theta\,d\theta = \ln\left|x + \sqrt{x^2-1}\right| - \frac{\sqrt{x^2-1}}{x} + C$, where I have skipped a few steps.

4. First calculate the decompostion $\frac{3x^2+1}{(x-1)(x^2+1)} = \frac{2}{x-1} + \frac{x+1}{x^2+1}$. Then $\int \frac{3x^2+1}{(x-1)(x^2+1)}\,dx =$
$\int \frac{2}{x-1}\,dx + \int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = 2\ln|x-1| + \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C =$
$\ln\left((x-1)^2\sqrt{x^2+1}\right) + \tan^{-1}x + C$.

5. $\int_3^{\infty} \frac{1}{x^2-x}\,dx = \lim_{b\to\infty}\int_3^b \frac{1}{x-1} - \frac{1}{x}\,dx = \lim_{b\to\infty}\left(\ln\frac{x-1}{x}\Big|_3^b\right) = \lim_{b\to\infty}\ln\frac{b-1}{b} - \ln\frac{2}{3} = \ln 3 - \ln 2 =$
$\ln\frac{3}{2}$, with either of the last two answers being correct. We have used the fact that $\ln\frac{b-1}{b} \to \ln 1 = 0$ as $b\to\infty$, the natural logarithm being continuous on $(0,\infty)$ and thus continuous at 1. Note that it would be incorrect to write the integral of the difference as the difference of the two integrals in the second step, because those two integrals individually diverge. This reflects the fact that the limit of a difference is the difference of the limits IF both individual limits exist–otherwise that can be false as it would be here. Specifically, the phrase $\infty - \infty$ is meaningless.

Class Statistics

Grade	Test#1	Test#2	Test#3	Test#4	Final Exam	Final Grade
90-100 (A)	15					
80-89 (B)	4					
70-79 (C)	3					
60-69 (D)	0					
0-59 (F)	0					
Test Avg	89.5%					