Print Your Name Here: ____________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 100 points.

1. (20 points) *Find all three first order partial derivatives of* $f(x, y, z) = \ln\left(x^2 + 2y + xyz\right)$.

2. (20 points) The length x and width y of a rectangle are measured at 10cm and 5cm respectively with a maximum error of 0.1cm for each measurement. *Use the differential* dA to estimate the error in the *calculated area* $A = xy$ of the rectangle.

3. (20 points) Let $F(x, y, z) = e^{xyz} + x + y + z$.

a. Find the gradient vector, $\nabla F(x, y, z)$.

b. Use the result of part (a) to find an equation for the tangent plane to the surface $F(x, y, z) = 3$ at the point $(1, 1, 0)$

.

4. (20 points) Let $f(x,y) = x^3 - \frac{3}{2}x^2 + y^3 - 3y$. Find all *four* critical points, and for each one *use the second derivative test* to determine whether it is a local maximum, a local minimum, or a saddle point.

5. (20 points) *Find the (absolute) maximum volume* of a rectangular box in the first octant with three faces in the coordinate planes, one vertex at the origin, and one vertex (x, y, z) *in the plane* $x + 2y + z = 2$. (Note that in the first octant we have $x \geqslant 0,\ y \geqslant 0$ and $z \geqslant 0$. The edge lengths of the box will be the coordinates, x, y and z, of the vertex lying in the plane $x + 2y + z = 2$.)

Solutions

1. $f_x = \dfrac{2x+yz}{x^2+2y+xyz}$, $f_y = \dfrac{2+xz}{x^2+2y+xyz}$, $f_z = \dfrac{xy}{x^2+2y+xyz}$. Compare with 14.3/33.

2. The area $A = xy$ and $dA = A_x(10,5)\Delta x + A_y(10,5)\Delta y = 5\Delta x + 10\Delta y = 1.5\text{cm}^2$. (You do not need to state the units.) Compare with 14.4/#33.

3. Compare with assigned homework problem 14.6/45.

- **a**. $\nabla F(x,y,z) = (yze^{xyz}+1,\ xze^{xyz}+1,\ xye^{xyz}+1)$.
- **b**. An equation for the tangent plane is $x+y+2z = 2$ or the equivalent. This is found by setting $\nabla F(1,1,0)\cdot(x-1, y-1, z-0) = 0$. A common error is to mingle part of the equation for the tangent plane to $z = f(x,y)$ with part of the equation for the tangent plane to an implicitly defined surface $F(x,y,z) = C$.

.

4. To find the critical points we find both first order partial derivatives $f_x = 3x(x-1)$ and $f_y = 3(y^2-1)$ and set them both equal to zero. The critical points are $(0,\pm1)$ and $(1,\pm1)$. To use the second derivative test, we must find $f_{xx} = 3(2x-1)$ and the discriminant $D = f_{xx}f_{yy} - f_{xy}^2 = 18y(2x-1)$. Saddle Points: (0,1), (1,-1). Local Maximum: (0,-1). Local Minimum: (1,1). See Fig. 1 below. Many errors were with algebra solving two equations in two unknowns. Compare with 14.7/#13.

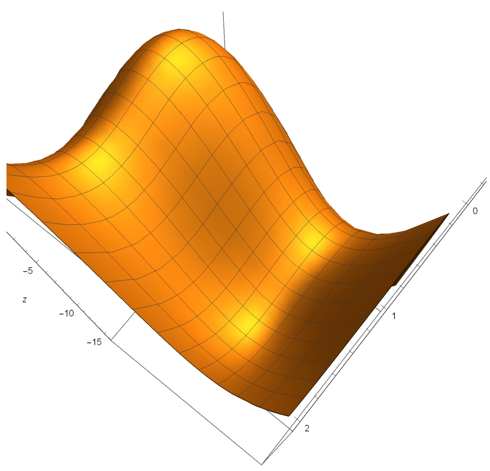

Figure 1: $f(x,y) = x^3 - \frac{3}{2}x^2 + y^3 - 3y$

5. For this problem the student must know that the *volume* $V = xyz = xy(2-x-2y)$ on the domain D bounded by the positive x and y axes and by the line $x+2y = 2$. Here we have expressed z in terms of x and y since the vertex (x,y,z) lies on the plane $x+2y+z = 2$. It is clear that $V = 0$ on the 3 edges of the triangular region D, so the maximum occurs at an interior point where V_x and V_y are both 0. It follows that at the maximum point $(x,y,z) = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$. Thus $V_{\max} = \frac{4}{27}$. Compare with 14.7/#49.

Problems #4 and #5 were the hardest for the class. Please compare your work on all five problems with the solutions to the corresponding problems in your homework notebook. And be sure to see me for help with any work that you need to understand better. Test grades in my classes are very rarely curved. In test #1 this year I applied a curve because I felt there was a preponderance of hard topics covered. *The grades recorded for Test #1 below were curved as follows: Take the raw score on the top of your test paper, multiply by 0.8 and then add 20 to find your curved grade.*

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	6				
80-89 (B)	20				
70-79 (C)	26				
60-69 (D)	19				
0-59 (F)	5				
This Test Avg	75.3%	%	%	%	%
Quiz Avg	67.8%	%	%	%	%