Print Your Name Here: ____________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 100 points.

1. (20 points) Use *Lagrange Multipliers* to find the *maximum value* and the *minimum value* of $f(x, y) = 2x + y$ on the ellipse $x^2 + 2y^2 = 18$.

2. (20 points) Evaluate the double integral $\iint\limits_R y\cos(xy)\,dA$ where $R = [0, \pi] \times [0, 1]$. Hint: One order of iteration is easier than the other.

3. (20 points) *Set up*, but do *not* evaluate, $\iint\limits_D y\,dA$ where D is the region of the xy-plane bounded by the parabola $x = y^2$ and the straight line $x = y + 6$. This means write the correct iterated integral in some order with lower and upper limits of integration that are correct for your chosen order of iteration. Suggestions: sketch the domain. One order of iteration is simpler than the other.

4. (20 points) Evaluate $\iiint\limits_R z\,dV$ where R is the tetrahedron bounded by the 3 coordinate planes and the plane $x + y + z = 1$. Suggestions: sketch the region and don't expand powers unless you must.

5. (20 points) Find the volume of the region R that lies above the cone $\phi = \dfrac{\pi}{4}$ and below the sphere $\rho = \cos\phi$.

Solutions

1. We set $\nabla f = \lambda \nabla g$ where $g(x, y) = x^2 + 2y^2$. This tells us that at an extreme point

$$\begin{aligned} 2 &= 2\lambda x & (1)\\ 1 &= 4\lambda y & (2)\\ x^2 + 2y^2 &= 18. & (3) \end{aligned}$$

Equations (1) and (2) imply that $xy\lambda \neq 0$. Thus $\frac{x}{2y} = 2$, which we substitute into (3) yielding $y = \pm 1$ and $x = \pm 4$. Hence the maximum value of f is 9 and the minimum is -9. Compare with 14.8 / 5.

2. $\iint\limits_R y\cos(xy)\,dA = \int_0^1 \int_0^\pi y\cos(xy)\,dx\,dy = \int_0^1 \sin(\pi y)\,dy = \frac{2}{\pi}$. Iteration in the opposite order is more work because it would entail integration by parts. Compare with 15/2 / 33.

3. $\iint\limits_D 1\,dA = \int_{-2}^{3} \int_{y^2}^{y+6} y\,dx\,dy$. If you chose the opposite order of iteration you would need two double integrals. Compare with 15.2 / 15.

4. $\iiint\limits_R z\,dV = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} z\,dz\,dy\,dx = \frac{1}{24}$. Compare with 15.6 / 15.

5. $V = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\cos\phi} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \frac{1}{3}\cos^3\phi \sin\phi\,d\phi\,d\theta = \int_0^{2\pi} -\frac{1}{12}\cos^4\phi \Big|_{\phi=0}^{\frac{\pi}{4}} d\theta = \frac{\pi}{8}$. Compare with 15.8 / 29.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	6	10			
80-89 (B)	20	22			
70-79 (C)	26	15			
60-69 (D)	19	11			
0-59 (F)	5	9			
This Test Avg	75.3%	76.4%	%	%	%
Quiz Avg	67.8%	65.6%	%	%	%
Quiz/Test Correl		0.84			

The correlation coefficient varies from -1 to 1, with correlations above 0.6 being considered strongly positive. The correlation coefficient in this class is the cosine of the angle between two data vectors in 78 dimensional Euclidean space–one dimension for each student enrolled.