Print Your Name Here: ______________________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 100 points.

1. (20) Find $\int_C xy^2\, ds$ where C is given by $x = \cos t, y = \sin t, 0 \leqslant t \leqslant \frac{\pi}{2}$.

2. (20) Let $\vec{F}(x,y) = 2xy^3\vec{i} + (3x^2y^2 + 2y)\vec{j}$.

a. (5) Show explicitly the equality of partial derivatives that establishes that $\vec{F}$ is conservative.

b. (10) Find a potential function $f(x,y)$ for $\vec{F}(x,y)$. Show your work!

c. (5) Use the result of part (b) to evaluate $\int_{(0,0)}^{(2,3)} \vec{F} \cdot d\vec{r}$, independent of path.

3. (20 points) Use Green's theorem to evaluate $\oint_C (y + \ln \sin x)\, dx + (2x + \ln \sin y)\, dy$ where C is the triangle with vertices $(0,0), (2,0)$ and $(0,3)$.

4. (20 points) Let S be the *upward oriented* paraboloid $z = x^2 + y^2$ lying above the disc D given by $x^2 + y^2 \leqslant 4$ in the xy-plane. Let $\vec{F}(x,y,z) = \langle x, y, 3z \rangle$. Find the *flux* ϕ of the vector field $\vec{F}$ across the surface S.

5. (20 points) Let S be the *upward oriented* hemisphere $z = \sqrt{4 - x^2 - y^2}$ which is bounded by a circle C in the xy-plane. Let $\vec{F}(x,y,z) = \langle ze^x,\ xy,\ xze^y \rangle$. *Use Stokes' theorem* to find $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$.

Solutions

1. $\int_C xy^2\,ds = \int_0^{\frac{\pi}{2}} \sin^2 t \cos t \sqrt{\sin^2 t + \cos^2 t}\,dt = \frac{1}{3}$. Compare with 16.2/9.

2. Compare with 16.3/13.

a. $M_y = 6xy^2 = N_x$.

b. $f(x,y) = x^2y^3 + y^2$ which is found by first setting $f_x = M$ and then substituting the most general solution into the condition $f_y = N$.

c. (5) $\int_{(0,0)}^{(2,3)} \vec{F} \cdot d\vec{r} = f(2,3) - f(0,0) = 117$.

3. $\oint_C (y + \ln \sin x)\,dx + (2x + \ln \sin y)\,dy = \iint_R N_x - M_y\,dA = \iint_R 2 - 1\,dA = 3$ where R is the region enclosed by the triangle C, the area of which is 3. This example illustrates how greatly Green's theorem can simplify a problem. Direct calculation of the line integral along C would be far more difficult. Compare with 16.4/7.

4. The upward normal $\vec{N} = \langle -2x, -2y, 1\rangle$ and, by definition, the flux $\phi = \iint_S \vec{F} \cdot \vec{n}\,dS =$ $\iint_D \vec{F} \cdot \vec{N}\,dA = \iint_D -2x^2 - 2y^2 + 3z\,dA = \iint_D x^2 + y^2\,dA = \int_0^{2\pi} \int_0^2 r^2\, r dr d\theta = 8\pi$. Compare with 16.7/23.

5. $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\,dS = \oint_C \vec{F} \cdot d\vec{r} = \oint_C ze^x\,dx + xy\,dy + xze^y\,dz = \oint_C xy\,dy = \int_0^{2\pi} 8\cos^2\theta \sin\theta\,d\theta =$ 0. Here we have made use of the fact that $z = 0$ everywhere along the curve C. (One could alternatively have used Green's theorem for the last oriented integral around C for the last step.) Compare with 16.8/3. Many students, despite repeated cautions about this in class, have ignored the instruction to use Stokes' theorem and instead attempted to calculate the flux of the curl field directly, resulting in a hopeless mess that demonstrates how useful Stokes' theorem is. That is why you are learning it in Math 2057 and that is why you were required to Use Stokes' Theorem so as to make your work so much simpler. *If you have not done so yet, please do so now, and also learn to use Green's theorem and the Divergence theorem before the Final Exam!*

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	6	10	11		
80-89 (B)	20	22	18		
70-79 (C)	26	15	14		
60-69 (D)	19	11	13		
0-59 (F)	5	9	10		
This Test Avg	75.3%	76.4%	74.5%	%	%
Quiz Avg	67.8%	65.6%	66.6%	%	%
Quiz/Test Correl		0.84	0.9		

The correlation coefficient varies from -1 to 1, with correlations above 0.6 being considered strongly positive. The correlation coefficient in this class is the cosine of the angle between two data vectors in 78 dimensional Euclidean space–one dimension for each student enrolled.