Print Your Name Here: ____________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 200 points.

1. (20) Let $F(x, y, z) = x^3y^2z$.

a. (10) Find $\nabla F(x, y, z)$.

b. (5) Find $\nabla F(1, 2, 3)$.

c. (5) Find an equation for the tangent plane to the level surface of F defined by $F(x, y, z) = 12$ at the point (1,2,3).

2. (20) Find all *four critical points* for $f(x, y) = x^3 - 3x + 3xy^2$. Use f_{xx} and the discriminant $D = f_{xx}f_{yy} - f_{xy}^2$ to classify each critical point as a *local maximum, local minimum,* or *saddle point.*

3. (20 points) *Find the point* (x, y, z) on the plane $x + 2y + 3z = 6$ that *minimizes* the value of $f(x, y, z) = \frac{1}{2}(x^2 + y^2 + z^2)$. Lagrange multipliers will be the simplest method.

4. (20 points) Evaluate $\displaystyle\iint_D y\, dA$ if D is the region bounded by the curves $y = x$ and $y = x^2$.

5. (20 points) Find the *surface area* of the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane.

6. (20) Evaluate $\displaystyle\iiint_E 1\, dV$ where E is the solid region bounded by the parabolic cylinder $y = x^2$ on the left side, the plane $z = 0$ below, and the plane $y + z = 1$ above and to the right.

7. (20) Use spherical coordinates to find $\displaystyle\iiint_E z\,dV$ where E is the region bounded above by the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$ and bounded below by the plane $z = 0$.

8. (20 points) Evaluate the line integral $\displaystyle\int_C x\,dy + y\,dz$ where C is the straight line segment from (0,1,2) to (2,4,6).

9. (20 points) *Use Green's theorem* to evaluate $\displaystyle\oint_C ye^x\,dx + 2e^x\,dy$ if C is the positively oriented perimeter of the rectangle $R = \{(x, y) \mid 0 \leqslant x \leqslant 4,\ 0 \leqslant y \leqslant 2\}$.

10. (20 points) Let the vector field $\vec{F}(x, y, z) = \langle x + y + e^x, y + z - ye^x, x^2 + y^2 + z\rangle$. *Use the Divergence theorem* to find $\displaystyle\iint_S \vec{F}(x, y, z) \cdot \vec{n}\,dS$, the flux of $\vec{F}$ across the outwardly oriented surface S of the rectangular box $E = \{(x, y, z) \mid 1 \leqslant x \leqslant 3,\ 2 \leqslant y \leqslant 4,\ 3 \leqslant z \leqslant 5\}$.

Solutions

1. Compare with 14.6/15, 43.

a. (10) $\nabla F(x,y,z) = \langle 3x^2y^2z, 2x^3yz, x^3y^2\rangle$.

b. (5) $\nabla F(1,2,3) = \langle 36, 12, 4\rangle = 4\langle 9,3,1\rangle$.

c. (5) $9(x-1) + 3(y-2) + z - 3 = 0$, or $9x + 3y + z = 18$.

2. $f_x = 3(x^2 + y^2 - 1)$ and $f_y = 6xy$. $f_x = 0$ on the circle $x^2 + y^2 = 1$ and $f_y = 0$ only on the two axes. The critical points are (0,1) and (0,-1) which are saddle points because $D = 36(x^2 - y^2)$ is negative there, and (1,0), a local minimum since $f_{xx} = 6x$ is positive there, and (-1,0), a local maximum. Compare with 14.7/11.

3. Let $g(x,y,z) = x+2y+3z$ and set $\nabla f = \lambda\nabla g$. It follows that $x = \lambda, y = 2\lambda$ and $z = 3\lambda$. Thus $x + 2y + 3z = 14\lambda = 6$. It follows that $(x,y,z) = \left(\frac{3}{7}, \frac{6}{7}, \frac{9}{7}\right)$. Compare with 14.7/41 and 14.8/35.

4. $\displaystyle\iint\limits_D y\,dA = \int_0^1\int_{x^2}^x y\,dy\,dx = \frac{1}{2}\int_0^1 y^2\Big|_{y=x^2}^x\,dx = \frac{1}{2}\int_0^1 x^2 - x^4\,dx = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{1}{15}$. Compare with 15.2/15.

5. $\displaystyle\mathcal{S} = \iint\limits_{x^2+y^2\leqslant 1} \sqrt{1 + 4(x^2+y^2)}\,dA = \int_0^{2\pi}\int_0^1 \sqrt{1+4r^2}\;r\,dr\,d\theta = 2\pi\frac{1}{8}\int_1^5 \sqrt{u}\,du = \frac{\pi}{4}\frac{2}{3}\,u^{\frac{3}{2}}\Big|_1^5 = \frac{\pi}{6}\left(5\sqrt{5} - 1\right)$. Compare with 15.5/5.

6. $\displaystyle\iiint\limits_E 1\,dV = \int_{-1}^1\int_{x^2}^1\int_0^{1-y} 1\,dz\,dy\,dx = \int_{-1}^1\int_{x^2}^1 (1-y)\,dy\,dx = \int_{-1}^1 \frac{1}{2}(1-x^2)^2\,dx = \frac{8}{15}$. Compare with 15.6/21.

7. $\displaystyle\iiint\limits_E z\,dV = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^1 (\rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{1}{4}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\phi\cos\phi\,d\phi\,d\theta = \frac{\pi}{4}$. Compare with 15.8/25.

8. We can parametrize C as $x = 2t$, $y = 3t+1$, $z = 4t+2$, $0 \leqslant t \leqslant 1$. Then $\displaystyle\int_C x\,dy + y\,dz = \int_0^1 2t\cdot 3\,dt + (3t+1)\cdot 4\,dt = \int_0^1 18t + 4\,dt = 13$. The parametrization is easily understood as representing a vector definition $\langle x,y,z\rangle = \langle 0,1,2\rangle + t\langle 2,3,4\rangle$, $0 \leqslant t \leqslant 1$. Compare with 16.2/15.

9. $\displaystyle\oint\limits_C ye^x\,dx + 2e^x\,dy = \iint\limits_R 2e^x - e^x\,dA = \int_0^4\int_0^2 e^x\,dy\,dx = 2e^x\big|_0^4 = 2(e^4 - 1)$. Compare with 16.4/5.

10. $\displaystyle\iint\limits_S \vec{F}(x,y,z)\cdot\vec{n}\,dS = \iiint\limits_E \nabla\cdot\vec{F}(x,y,z)\,dV = \int_1^3\int_2^4\int_3^5 1 + e^x + 1 - e^x + 1\,dz\,dy\,dx = \int_1^3\int_2^4\int_3^5 3\,dz\,dy\,dx = 24$. Compare with 16.9/5.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	6	10	11	21	13
80-89 (B)	20	22	18	28	26
70-79 (C)	26	15	14	9	16
60-69 (D)	19	11	13	8	10
0-59 (F)	5	9	10	7	8
This Test Avg	75.3%	76.4%	74.5%	82.4%	79.1%
Quiz Avg	67.8%	65.6%	66.6%	66.6%	66.6%
Quiz/Test Correl		0.84	0.9	0.7	0.7
Attendance/Test Correl				0.63	0.63

The correlation coefficient varies from -1 to 1, with correlations above 0.6 being considered strongly positive. The correlation coefficient in this class is the cosine of the angle between two data vectors in 78 dimensional Euclidean space–one dimension for each student enrolled. The two correlation coefficients shown indicate that the final test average in the course has a strongly positive correlation with both attendance and performance on the homework quizzes.