Print Your Name Here: _____

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 100 points.

1. (30 points)

a. (20) If R is the *first quadrant* region between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$, rewrite the integral $\iint_R \sin(x^2 + y^2)\, dA$ in polar coordinates. (Be sure to include the lower and upper limits on the iterated integrals.)

b. (10) Evaluate the integral in polar coordinates from part (a).

2. (20 points) Set up, but DO NOT EVALUATE, the triple integral in *rectangular coordinates* for $\iiint_E y^2\,dV$ if E is the tetrahedron with the vertices $(0,0,0)$, $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$, being sure to find the correct lower and upper limits for each of the iterated integrals.

3. (20 points) Rewrite the integral $\iiint_E (x+y+z)\,dV$ in *cylindrical* coordinates, if E is the *first octant* solid bounded above by the paraboloid $z = 4 - x^2 - y^2$. Be sure to include all lower and upper limits of integration , but, to save time, DO NOT EVALUATE the integral.

4. (30 points) Let E denote the region between the two spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$.

a. (20) Set up the triple integral in spherical coordinates for $\iiint_E (x^2 + y^2)\, dV$. (Include the lower and upper limits for each iterated integral in the order of iteration you've chosen.)

b. (10) Evaluate the integral from part (a). (*No decimal approximations!*) (Hint: Recall that $\sin^2\phi$ can be expressed in terms of $\cos^2\phi$.)

Solutions

Please read and follow the instructions on page 1. Please learn to write your solutions with valid equations, which are mathematical statements. And the symbol "=" has a precise meaning which must be used properly. And a sound solution is not a mere sequence of steps. It is a logically ordered sequence of mathematical statements, in which each statement follows from what came before. Remember to use correct mathematical punctuation such as parentheses, brackets, or braces. The order of operations is important.

1. (30) This was problem 15.3/11. Do not overlook the italicized *first quadrant*!

a. (20) $\iint_R \sin(x^2 + y^2)\, dA = \int_0^{\frac{\pi}{2}} \int_1^3 \sin(r^2)\, r dr\, d\theta$. Please note: $\theta \neq \phi$. These are different Greek letters.

b. (10) $\int_0^{\frac{\pi}{2}} \int_1^3 \sin(r^2)\, r dr\, d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} \int_1^9 \sin u\, du\, d\theta = \frac{\pi}{4}(\cos 1 - \cos 9)$, where $u = r^2$.

2. (20) $\int_0^2 \int_0^{2-x} \int_0^{2-x-y} y^2\, dz\, dy\, dx$ or an equivalent formulation with correct lower and upper limits corresponding to your chosen order of iteration. The tilted plane is given by $x + y + z = 2$. Some students drew a tetrahedron correctly but used the limits of integration for a cube! That makes no sense at all. This was problem 15.6/19.

3. (20) $\iiint_E (x + y + z)\, dV = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} (r\cos\theta + r\sin\theta + z)\, r dz\, dr\, d\theta$ This was part of problem 15.7/21.

4. (30) This is similar to 15.8/25, but we have used easier choices of the two radii to simplify the arithmetic.

a. (20) $\int_0^{\pi} \int_0^{2\pi} \int_1^2 (\rho\sin\phi)^2(\cos^2\theta + \sin^2\theta)\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi = \int_0^{\pi} \int_0^{2\pi} \int_1^2 (\rho\sin\phi)^2 \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi = \int_0^{\pi} \int_0^{2\pi} \int_1^2 \rho^4 \sin^3\phi\, d\rho\, d\theta\, d\phi$, or an equivalent form.

b. (10) $\int_0^{\pi} \int_0^{2\pi} \int_1^2 \rho^4 \sin^3\phi\, d\rho\, d\theta\, d\phi = \frac{248}{15}\pi$. To anti-differentiate $\sin^3\phi$, write $\sin^3\phi = (1 - \cos^2\phi)\sin\phi$ and make a simple substitution.

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	4	17			
80-89 (B)	9	4			
70-79 (C)	9	5			
60-69 (D)	4	3			
0-59 (F)	5	4			
This Test Avg	75.5%	80.8 %	%	%	%
HW Avg	95.1 %	92.6 %	%	%	%
HW/TST Corr	0	0.37			

The Correlation Coefficient is the cosine of the angle between two data vectors in $\mathbb{R}^{33}$–one dimension for each student enrolled. Thus this coefficient is between 1 and -1, with coefficients above 0.6 being considered strongly positive. The correlation coefficient shown indicates that the test grades in the course have a moderately positive correlation with performance on the graded homework. In geometrical terms, the test vector makes an angle of 1.19 radians (about 68 degrees) with the graded homework vector. My intuition is that more students are now doing their own homework instead of farming it out to AI.