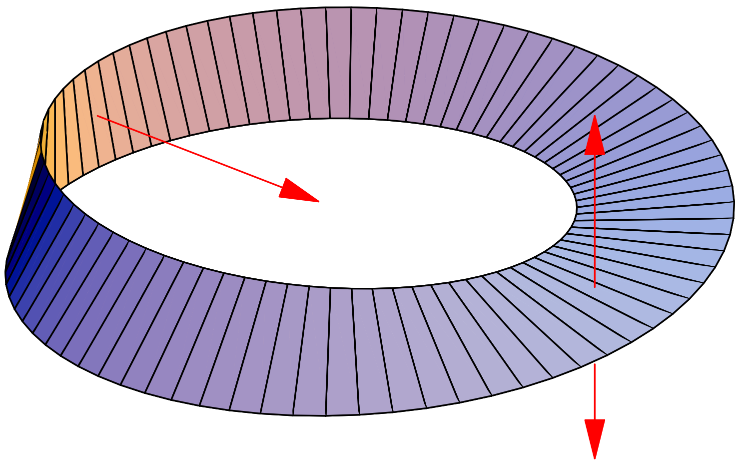

Imagine designing a paper air filter for an engine, or a wire-mesh fuel filter. We can imagine this as being a surface with two sides. One side faces towards the engine and the other faces away. We are interested in the flow of air or fuel through the surface in the direction towards the engine. In order to have a surface which is capable of functioning as a filter, it is necessary to be able to pick two distinct sides of the surface. This might seem easy, but in fact it is not always possible!

The *Möbius band* is an example of a *one-sided surface*. Here are several ways to picture what this means.

- Imagine walking along this strip, never crossing an edge. Do this like a fly so you don't fall off! Is there any part of the surface you do not cover?
- Is it possible to select one side over another? Look at the upright red arrow. If you cut a small disk out of the surface around the base of the arrow, you could paint one side blue and the opposite side red, for example.

There are two sides of the disk. But something remarkable happens when we consider the surface as a whole. Try painting blue the side on which you see the base of the upright red arrow, and continue painting, never crossing an edge. What happens? Do you see that you wind up painting the entire surface, and nothing remains to be painted red?

- Equivalently, try to select a continuously varying unit normal everywhere on this surface? Begin with the upright red vector, and imagine this vector walking along the surface, not falling off. Remember that a surface is only one point thick! When we reach the point at which the red vector points straight down, we are at the same point where the vector had stood straight up. We cannot select a continuously varying unit normal over the entire surface.

- We can think of these upright red arrows as designating a direction of flow through a surface, as with a filter for air or fuel. The two opposing arrows show us that we cannot use a *Möbius band* band for a filter: It has only one side, in a global sense.

Because we cannot select an *orientation* for this surface, it is called *non-orientable.* Thus the concept of the flux of a vector field is meaningless for this surface, and Stokes' Theorem does not apply to the *Möbius band*.

It is entertaining also to imagine running a finger along an edge of this surface without removing it from the edge. How many edges does this surface have?