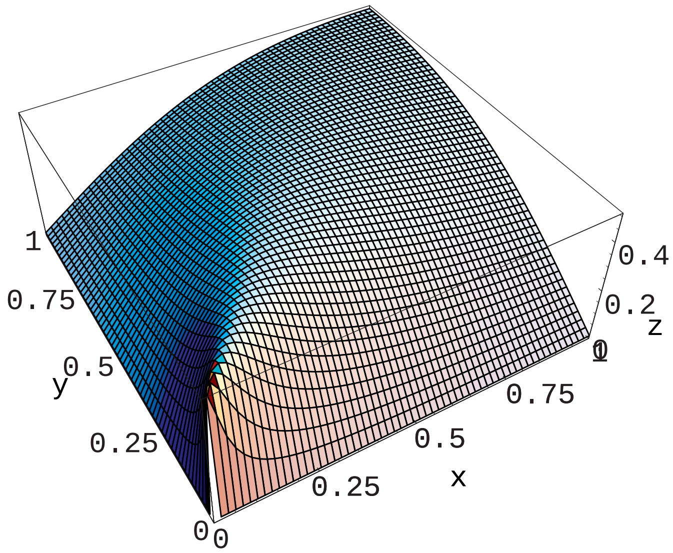


Here

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

In this picture x and y vary from 0 to 1. Show that

$$f_x(0,0) = 0 = f_y(0,0),$$

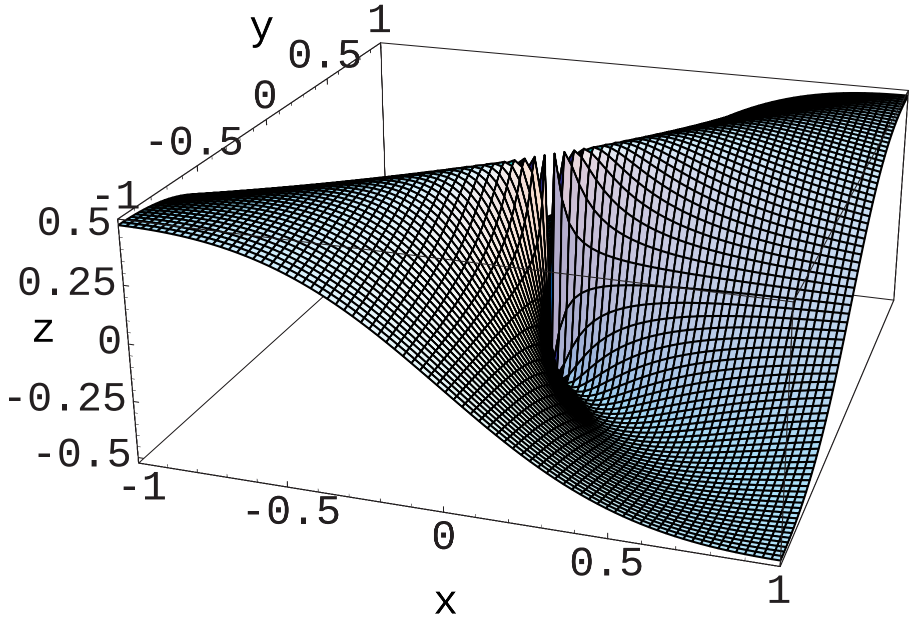
yet f is *not* differentiable at $(0,0)$. What happens as $(x,y) \to (0,0)$ along the line $y = x$?

Below we see a larger view of the graph of this same function, except with $|x| \leq 1$ and $|y| \leq 1$. Can you see where the origin is? What does $f(x,y)$ approach as $(x,y) \to (0,0)$ along the line $y = -x$?

In the text by Prof. Rogawski, **Theorem 2** of **Section 14.5**, and **Theorem 1** of **Section 14.6**, state a special and a general version of the **Chain Rule**. However, the rule as stated is *incomplete*. It states and proves only that the composition of two differentiable functions has partial derivatives.

A full and much better statement of the Chain Rule asserts that *the composition of two differentiable functions is differentiable.* The Example presented above shows that the *existence of partial derivatives establishes neither differentiability nor continuity.*

The student needs to be aware that in fact the composition of two differentiable functions must be differentiable, and this is the strongest and most important conclusion of the Chain Rule. The proof is very similar to the proof that the partial derivatives exist, though it is a bit more intricate. You may see me for further details.



y
1
0.5
0
-0.5
-1
0.5
0.25
z
0
-0.25
-0.5
-1
-0.5
0
0.5
1
x