



In this Example, we are asked to plot the vector field  $\vec{F}(x, y) = y\vec{i} + \frac{1}{2}\vec{j}$ . We will see that this vector field consists of the tangents to a certain family of parabolas.

We seek a curve  $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$  such that

$$\vec{r}'(t) = x'(t)\vec{i} + y'(t)\vec{j} = y(t)\vec{i} + \frac{1}{2}\vec{j}$$

Setting  $y'(t) = \frac{1}{2}$  we see that  $y(t) = \frac{t}{2} + C$ . Then we set  $x'(t) = y(t) = \frac{t}{2} + C$  so that  $x(t) = \frac{t^2}{4} + Ct + D$ . Actually, we don't need so many parameters to get parabolas which collectively fill the whole  $xy$ -plane. We set  $C = 0$  and let  $D$  vary over the real numbers. In the picture above we show the parabolas for  $D = 0, -1, -2, -3$  and we can see that the vector field is tangent to the parabolas. (The vectors have been *scaled* so that the maximum length is one.) Such curves as these are called *integral curves* for the given vector field. We will not study integral curves in this course, but integral curves are very important in the study of *differential equations*.