In this Example, we are asked to plot the vector field \( \vec{F}(x, y) = y\vec{i} + \frac{1}{2}\vec{j} \). We will see that this vector field consists of the tangents to a certain family of parabolas.

We seek a curve \( \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} \) such that
\[
\vec{r}'(t) = x'(t)\vec{i} + y'(t)\vec{j} = y(t)\vec{i} + \frac{1}{2}\vec{j}
\]

Setting \( y'(t) = \frac{1}{2} \) we see that \( y(t) = \frac{t}{2} + C \). Then we set \( x'(t) = y(t) = \frac{t}{2} + C \) so that \( x(t) = \frac{t^2}{4} + Ct + D \). Actually, we don’t need so many parameters to get parabolas which collectively fill the whole \( xy \)-plane. We set \( C = 0 \) and let \( D \) vary over the real numbers. In the picture above we show the parabolas for \( D = 0, -1, -2, -3 \) and we can see that the vector field is tangent to the parabolas. (The vectors have been scaled so that the maximum length is one.) Such curves as these are called integral curves for the given vector field. We will not study integral curves in this course, but integral curves are very important in the study of differential equations.