Name:

Instructions. Show all work in the space provided. Indicate clearly if you continue on the back side, and write your name at the top of the scratch sheet if you will turn it in for grading. No books or notes are allowed, but a scientific calculator is ok - though unnecessary. All work must be shown to receive credit. Please do not give decimal approximations for square roots, for trigonometric or exponential functions, or for $\pi$. Maximum score = 100 points.

1. (20) Evaluate the line integral $\int_C xy\,dx + (x - y)\,dy$ where $C$ is the straight line segment from $(2,0)$ to $(3,2)$.

2. (20) Find a potential function $f$ and use it to evaluate the path-independent integral

$$\int_{(0,0)}^{(1,2)} (y^2 + 2xy)\,dx + (x^2 + 2xy + y)\,dy$$
3. (20) Use Green’s Theorem to evaluate

\[ \oint_C (y + e^{-x^2/2})dx + 2xdy \]

if \( C \) is the positively oriented curve consisting of the interval \([0, \pi]\) on the \( x \)-axis together with the graph of \( y = \sin x \), \( 0 \leq x \leq \pi \). (Don’t be alarmed - just apply Green’s Theorem!)

4. (20) Let \( \vec{F}(x, y, z) = 2y\vec{i} + 3z\vec{k} \), and let \( \mathcal{S} \) be the surface \( x = u, y = v, z = 3u + 2 \) with \( 0 \leq u \leq 1, 0 \leq v \leq 1 \). Let \( \vec{n} \) be the upward unit normal to \( \mathcal{S} \). Find \( \iint_S \vec{F} \cdot \vec{n} dA \).
5. (10) Let $R$ be the box $|x| \leq 1, |y| \leq 2, |z| \leq 3$. Use the Divergence Theorem to find 
\[ \iiint_{\partial R} \vec{r} \cdot \vec{n} \, dA, \] 
where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ is the position vector, and $\vec{n}$ is the outward unit normal.

6. (10) Let $S$ be the surface given by $\vec{r} = u\vec{i} + v\vec{j} + (1 - 3u - v)\vec{k}, 0 \leq u \leq 1, 0 \leq v \leq 2$, with upward unit normal, as seen from the positive z-axis. Let

\[ \vec{F}(x, y, z) = (x - z)\vec{i} + (x + y)\vec{j} + (y - z)\vec{k} \]

Use Stokes’ Theorem to evaluate $\int_{\partial S} \vec{F} \cdot d\vec{r}$. 