Much Improved Version of Sec. 1.5 / Exercise 5: Use the following steps to prove that *every* sequence $x_n$ of real numbers has a monotone subsequence. Denote the $n^{th}$ tail of the sequence by $T_n = \{x_j \mid j \geq n\}$.

- (a) Suppose the following special condition is satisfied: For each $n \in \mathbb{N}$, $T_n$ has a smallest element. Prove that there exists an increasing subsequence $x_{n_j}$.

- (b) Suppose the condition above fails, so that there exists $N \in \mathbb{N}$ such that $T_N$ has no smallest element. Prove that there exists a decreasing subsequence $x_{n_j}$. 