Results

- Classified up to diffeomorphism all 4–manifolds which admit a symplectic form and a circle action with at least one fixed point. (Note that this is different than classifying 4–manifolds with symplectic circle actions.)

  - Most theorems about smooth 4–manifolds describe obstructions to two manifolds being diffeomorphic. This positive theorem says that if a 4–manifold has a symplectic form and a circle action with fixed points than it must be one of a finite number of example types.

  - Used this result to prove a conjecture which is a generalization of Taubes’s question “If $Y^3 \times S^1$ is symplectic, does $Y$ fiber over $S^1$?” when the 4–manifold has an effective circle action with at least one fixed point. The full conjecture is:

    **Conjecture.** Every symplectic 4–manifold which admits an effective circle action also admits a (possibly different) symplectic form and effective circle action such that the circle action is symplectic with respect to the form.

  - This classification implies that the only 4–manifolds which admit a symplectic form and a circle action with at least one fixed point must have $b_+ = 1$.

  - Links two related yet distinct research groups together: those who study the global invariants of symplectic manifolds and those who study manifolds with symplectic group actions.

- Produced an infinite family of new symplectic 4–manifolds with Kodaira dimension 1, $b_+ = 1$, $b_1 = 2$, and not of Lefschetz type. Such examples were not known to exist. This has been an open question since at least 1995.

  - These manifolds have very special properties — they are not complex manifolds, their Seiberg-Witten invariants are independent of the chamber structure, and they do not have metrics of positive scalar curvature.

  - These examples can be used to completely fill out the symplectic geography of symplectic manifolds with Kodaira dimension 1 where one also takes into account the rank of the kernel of the map $\cup [\omega] : H^1(X; \mathbb{R}) \to H^3(X; \mathbb{R})$. (See research statement.)

- Provided examples of 3–manifolds for which the Milnor torsion differs from Turaev torsion. For these manifolds Milnor torsion is identically zero while Turaev torsion is not.

- Discovered formulas for calculating the Seiberg-Witten invariant of any smooth 4–manifold with $b_+ > 0$ and an effective circle action.

  - When the action is fixed point free the formulas relate (generally hard to calculate) diffeomorphism invariants on the 4–manifold with easy to calculate Milnor torsion on the quotient 3–manifold.
– When $b_+ > 1$ and the action has a fixed point, the Seiberg-Witten invariant vanishes identically. Vanishing theorems of this type are useful for calculating relative Seiberg-Witten invariants.

– The proof of the formulas imply that 4–manifolds with $b_+ > 1$ and effective circle actions satisfy the Seiberg-Witten simple type conjecture.

– The main step in the proof of the formula for fixed point free circle actions was to show that the moduli space of solutions to the Seiberg-Witten equations on the 4–manifold was diffeomorphic to the moduli space of solutions on the quotient 3–manifold in a suitable sense.

• Produced an example of a non-symplectic 4–manifold with a free circle action whose quotient fibers over the circle.

– It was known that most circle bundles over a fibered 3–manifold were symplectic, but it was not known if there existed a circle bundle with $b_+ > 0$ over a fibered 3–manifold which was not symplectic.

– Example suggests a theorem that specifies exactly which circle bundles over a fibered 3–manifold can be symplectic. (See research statement.)

• Found easy to use formulas for calculating the 3-dimensional Seiberg-Witten invariants of the total space of a circle bundle over a surface.

• Defined Seiberg-Witten orbifold invariants for 3–manifolds with mild singularities. (This suggests that there are torsion invariants for orbifolds, see research statement.)

Citations

Although my papers are only a couple of years old, they are starting to be cited. Here are some examples:


• (For a main result) T. Etgü, Lefschetz fibrations, complex structures and Seifert fibrations on $S^1 \times M^3$, Algebr. Geom. Topol. 1 (2001), 469–489.

• (For a technical result) E. Lerman, S. Tolman, and C. Willett, SU(2) Invariant Contact Manifolds, preprint.