Comments concerning $P[X < t]$ and $P[X < t]$

The DISTRIBUTION FUNCTION $F_X$ for a random variable $X$ is defined by

$$F_X(t) = P[X \leq t]$$

Observe that we have here $\leq$, not $<$. The probability of the event $[X < t]$ can be obtained by as a limit:

$$P[X < t] = \lim_{x \to t^-} P[X < t] = \lim_{x \to t^-} F_X(t)$$

If the function $F_X$ happens to be continuous at $t$ then, and only then, $P[X < t]$ is equal to $F_X(t)$.

Example.
Consider the distribution function given by

$$F_X(x) = \begin{cases} 
0 & \text{if } x < -1 \\
\frac{1}{3} + \frac{2}{3}(x + 1)^2 & \text{if } -1 \leq x \leq 0
\end{cases}$$

The values of $F_X(x)$ for $x > 0$ are not stated. This is because

$$F_X(0) = \frac{1}{3} + \frac{2}{3} = 1$$

which means $P[X \leq 0] = 1$, and so $F_X(x) = 1$ for all $x \geq 0$.

Observe also that

$$P[X \leq -1] = F_X(-1) = \frac{1}{3} + 0 = \frac{1}{3}$$

while, on the other hand, $P[X < x] = F_X(x) = 0$ for any $x < 0$. This implies that

$$P[X = -1] = \frac{1}{3}$$

What is happening here is that the function $F_X$ has a discontinuity at $-1$.

Let us calculate $P[|X - \frac{1}{3}| < 1]$:

$$P \left[ \left| X - \frac{1}{3} \right| < 1 \right] = P \left[ -\frac{2}{3} < X < \frac{4}{3} \right] = P \left[ X < \frac{4}{3} \right] - P \left[ X \leq -\frac{2}{3} \right] = 1 - F_X \left( -\frac{2}{3} \right) = 1 - \left[ \frac{1}{3} + \frac{2}{3} \frac{1}{3^2} \right]$$