1. Let $\sigma \in S_5$ be the permutation given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 3 & 2 \end{pmatrix}$$

(i) Write down the value of $\sigma(2)$

(ii) Write down the permutation $\sigma^{-1}$.

(iii) Work out the permutation $\sigma^2$. 

(iv) Write down the cycle decomposition of $\sigma$. (4 pts)

(v) Work out, with explanation, $\text{sgn}(\sigma)$. (2 pts)

(vi) Write down a transposition that does not commute with $\sigma$. (2 pts)

(vii) Express the cycle (135) as a product of transpositions. (2 pts)
2. Suppose $G$ is a group in which the square of every element is the identity. Prove that $G$ is commutative. (5pts)
3. Let $G$ be a group of prime order (i.e. the number of elements in $G$ is a prime number). Prove that $G$ is cyclic. (5pts)