1. Suppose $H$ and $K$ are finite subgroups of a group $G$ such that $|H|$ and $|K|$ are coprime. Prove that $H \cap K = \{e\}$, where $e$ is the identity element in $G$. (10pts)
2. Write out all the six elements of $S_3$ and then write out all possible subgroups of $S_3$. (10pts)
3. Recall that for any integer $m$, we have the quotient $\mathbb{Z}_m = \mathbb{Z}/m\mathbb{Z}$ which is a group under addition. Elements of $\mathbb{Z}_m$ are the cosets $a + m\mathbb{Z}$ with $a$ running over all integers. On $\mathbb{Z}_6$ consider the mapping given by

$$f : \mathbb{Z}_6 \rightarrow \mathbb{Z}_6 : x + 6\mathbb{Z} \mapsto 3x + 6\mathbb{Z}$$

This is well-defined and is a homomorphism. Write out the kernel of $f$ and the image of $f$. 

(10pts)
4. Let $G$ be a group, $x$ an element of $G$, and let $C_x$ be the set of all $y \in G$ which commute with $x$, i.e.

$$C_x = \{ y \in G : yx = xy \}$$

Verify that $C_x$ is a subgroup of $G$. (10pts)