Homework 1

1. Let X be a non-empty set, and $\mathcal{F}$ a σ–algebra of subsets of X. Assume that $\mathcal{F}$ is finite, i.e. contains finitely many subsets of X. For each $x \in X$, let $\mathcal{F}_x = \{A \in \mathcal{F} | x \in A\}$, and $A_x = \cap \mathcal{F}_x$, the intersection of all the sets of $\mathcal{F}_x$. The set A_x will be called *the atom* of $\mathcal{F}$ at the point x.
   (i) Show that $A_x \in \mathcal{F}$.
   (ii) If $x \neq y$ show that either $A_x = A_y$ or A_x is disjoint from A_y.
   (iii) Show that each set in $\mathcal{F}$ is a union of atoms.
   (iv) Let $\mathcal{A}$ be the set of all atoms. Show that there is a bijection from $\mathcal{F}$ onto $2^{\mathcal{A}}$, where $2 = \{0, 1\}$.
   (v) Show that there is no σ–algebra of subsets of X which contains a countable infinity of sets.[Hint: Do (i)-(iv) under the assumption that $\mathcal{F}$ is countably infinite and arrive at a contradiction in (iv).]