

MATHEMATICS COLLOQUIUM

Tuesday, November 14, 2006
3:40-4:30 PM Lockett 285

Combinatorial aspects of the heat kernel measure on the unitary group

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Computing asymptotic quantities related to the heat kernel measure on the unitary group $U(N)$ as N tends to infinity is one of the basic questions of large N Yang-Mills theory, and a natural problem in the framework of large random matrices. P. Biane (1995) and F. Xu (1997) have independently computed limiting distributions and proved asymptotic freeness results. Recently, A. Sengupta has reformulated Xu's computation of the limiting distribution in a very clear and attractive way. In this talk, I will explain how physical ideas related to "string theories" developed in the context of two-dimensional Yang-Mills theory by D. Gross and W. Taylor (1993) shed some light on the approach of Xu and Sengupta, in particular on its combinatorial aspects.

More concretely, I will explain how elementary computations related to the Schur-Weyl duality allow one to relate the Brownian motion on the unitary group and the most natural random walk on the symmetric group. Then I will derive and discuss convergent series expansions for expectations of products of traces of unitary matrices under the heat kernel measure. This discussion will involve paths in the Cayley graph of the symmetric group and the lattice of non-crossing partitions.

Refreshments will be served in the lounge at 3pm
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