First Annual

TX-LA SIAM Sectional Meeting

at Louisiana State University, Baton Rouge October 5–7, 2018

LIST OF ABSTRACTS

Plenary presentations

Computational Models of Microscopic Fluid Flows with Biological Applications Ricardo Cortez, Mathematics Department Tulane University

The term Biofluid dynamics refers to the motion of fluids generated in biological and biomedical phenomena. These include blood flow in capillaries, the flows surrounding moving cells, bacteria, spermatozoa, and more. Many of these phenomena involve a flexible, elastic body exerting forces to the fluid in order to propel itself or generate flows that are advantageous in some way. These phenomena are of interest to scientists who study how the fluid environment and the forces generated on the elastic bodies combine to produce organism behaviors observed in the laboratory. I will present the main ideas behind a computational model called "the method of regularized Stokeslets" that is widely used to study microscopic flows generated by flagella (like those attached to bacteria and sperm cells). The presentation will be based on vector calculus and introductory partial differential equations. I will show simulations of flagellar motions that aim to understand the effect of asymmetry in the flagellar beat patterns as well as interactions with a nearby surface, which is important in fertilization.

Mathematical Methods for Next Generation Stent Design

Suncica Canic, Departments of Mathematics at UC Berkeley and University of Houston

Real life problems often times give rise to problems whose understanding requires development of sophisticated mathematical methods. In particular, mathematical methods for next generation stent design require novel results in nonlinear analysis of partial differential equations, moving boundary problems, and numerical methods for multiphysics, multiscale problems defined on moving domains. In this talk we will review the state of the art in mathematical methods for next generation stent design. Stents are mesh like devices used in the treatment of various cardiovascular diseases. Currently used stents include bare metal stents, biodegradable stents, and drug eluting stents. Despite the beneficial effects of stenting, persistent high rates of complications such as in stent restenosis and late stent thrombosis call for novel approaches to stent design. Recent ideas based on nanoengineered stents seem to be particularly promising. They include: (1) nano engineered, drug free stents, which are stents covered with nano engineered surfaces that promote accelerated restoration of functional endothelium and provide a drug free approach to keeping stents patent long term; and (2) ferromagnetic stents with magnet enhanced nano particle drug delivery of Anti thrombogenic drugs, or viable endothelium, for improved arterial wall healing. We will show how the following mathematical results have and will continue to influence further development of these bioengineering inventions: existence results in nonlinear fluid structure interaction (FSI), FSI with novel multi layered poroelastic structures, an extension of the Lions Aubin Simon compactness theorem to problems on moving domains, development of loosely coupled partitioned schemes for FSI problem, theory of 1D hyperbolic nets obtained using dimension reduction to model stents mechanical properties, and Smoothed Particle Hydrodynamics approaches to study emergent behaviors of active ferromagnetic nano particles. This is a joint work with post doc Y. Wang (Berkeley), professors J. Tambaca, B. Muha (U of Zagreb, Croatia), M.Bukac (Notre Dame), interventional cardiologist D. Paniagua (Baylor College of Medicine, and biomedical engineer Prof. T. Desai (UCSF).

Supported by the National Science Foundation, the LSU Mathematics Department, Louisiana State University, SIAM staff, and participant fees

Mixed Methods for Two-Phase Darcy-Stokes Mixtures of Partially Melted Materials with Regions of Zero Porosity

Todd Arbogast, University Texas, Austin

The Earth's mantle, or an ice sheet, involves a deformable solid matrix phase within which a second phase, a fluid, may form due to melting processes. The mechanics of this system is modeled as a dual-continuum, with at each point of space the solid matrix being governed by a Stokes system and the fluid melt, if it exists, being governed by a Darcy law. This system is mathematically degenerate when the porosity (volume fraction of fluid) vanishes. We develop the variational framework needed for accurate approximation of this Darcy-Stokes system, even when there are regions of positive measure where only one phase exists. We then develop an accurate mixed finite element method for solving the system and show numerical results that illustrate its convergence behaviour and performance. This is joint with Marc A. Hesse and Abraham L. Taicher.

Minisymposia

Nonlinear Partial Differential Equations and Applications, Part I Organizers: Phuc Cong Nguyen and Karthik Adimurthi

This mini-symposium is meant to bring in researchers working with nonlinear and quasilinear partial differential equations, fractional operators, potential theory and parabolic evolution equations. Many of these equations arise while studying natural phenomenon occurring in nature and understanding qualitative and quantitative aspects of solutions of these equations is the purpose of the mini-symposium.

Periodic Homogenization of Quasilinear Elliptic Equations

Jinping Zhuge, University of Kentucky

We study the convergence rates and uniform regularity in periodic homogenization of quasilinear elliptic equations. Under a general framework, we obtain the power rates of convergence in L^2 for the weak solutions in Lipschitz and $C^{1,\alpha}$ domains with different strategies. With these, the large-scale Lipschitz estimate up to boundary is established. Finally, we introduce a new approach involving a real-variable argument to establish the $W^{1,p}$ estimate in Reifenberg domains with $2 \le p < \infty$ in the context of periodic homogenization.

Delta Shock Formation in the $N \times N$ Keyfitz and Kranzer System Ralph Saxton, University of New Orleans

We examine the Cauchy problem for the system of conservation laws $U_t + (\Phi(U)U)_x = 0$, $U : R_t \times R_x \to R^n$, $n \ge 2$, where $\Phi(U) = \phi(r, \Theta) : R^n \to R$, r = |U| and $\Theta = U/|U| \in S^{n-1}$. We find a wide class of functions ϕ for which the amplitude of solutions can either blow up in finite time on sets of positive measure, or delta shock solutions can form, provided $\nabla_{\Theta}\phi \neq 0$ in regions of phase space where strict hyperbolicity fully fails. Joint work with Katarzyna Saxton.

Boundary value problems for higher order elliptic differential equations Ariel Barton, University of Arkansas

Second order differential operators of the form $\nabla \cdot A \nabla$, with A a uniformly positive definite matrix, have been studied extensively. Higher order operators of the form $\nabla^m \cdot A \nabla^m$ are much less well understood. In this talk I will establish well posedness of certain boundary value problems for higher order operators in divergence form.

Qualitative analysis of equations associated to sharp Hardy-Sobolev and Caffarelli-Kohn-Nirenberg inequalities

John Villavert, University of Texas, Rio Grande Valley

We discuss some recent results on the qualitative and quantitative properties of solutions to a nonlinear equation arising from several fundamental problems in analysis, geometry and the physical sciences. Namely, we shall describe several key results on positive solutions of the equation, which includes the necessary and sufficient conditions governing their existence as well as results on their asymptotic behavior and classification. Moreover, we also discuss applications of these results to the aforementioned fundamental problems.

Nonlinear Partial Differential Equations and Applications, Part II

Organizers: Phuc Cong Nguyen and Karthik Adimurthi

This mini-symposium is meant to bring in researchers working with nonlinear and quasilinear partial differential equations, fractional operators, potential theory and parabolic evolution equations. Many of these equations arise while studying natural phenomenon occurring in nature and understanding qualitative and quantitative aspects of solutions of these equations is the purpose of the mini-symposium.

A Potential space estimate for solutions of system of coupled nonlocal equations Tadele Mengesha, University of Tennessee

We will discuss that weak solutions to the strongly-coupled system of nonlocal equations of linearized peridynamics belong to a potential space with higher integrability. Specifically, we show a function that measures local fractional derivatives of weak solutions to a linear system belongs to L^p for some p that is larger than 2 with no additional assumption other than measurability and ellipticity of coefficients. This is a nonlocal analogue of an inequality of Meyers for weak solutions to an elliptic system of equations. We also show that functions in L^p whose Marcinkiewicztype integrals are in L^p do in fact belong to a Bessel potential space.

Quasilinear elliptic equations with weights

Cao Dat, Texas Tech University

We study the degenerate/singular quasi- linear elliptic equations in the sub-critical case. Existence and pointwise estimates of solutions are presented. Necessary and sufficient conditions for the existence of solutions in weighted Sobolev spaces are obtained. Our approach is based on the use of "weighted-Wolff" potentials and related integral inequalities.

Quantitative uniqueness of partial differential equations

Jiuyi Zhu, Louisiana State University

Motivated by the study of eigenfunctions, we consider the quantitative uniqueness of partial differential equations. The quantitative uniqueness is characterized by the order of vanishing of solutions, which describes quantitative behavior of strong unique continuation property. Strong unique continuation property states that if a solution that vanishes of infinite order at a point vanishes identically. It is interesting to know how the norms of the potential functions and gradient potentials control the order of vanishing. We will report some recent progresses about quantitative uniqueness in different spaces for elliptic equations and parabolic equations. Part of this work is joint with Blair Davey.

A unified approach to parabolic quasilinear equations

Karthik Adimurthi, Seoul National University, South Korea

We shall be interested in the Sobolev properties of Quasilinear parabolic problems with either constant or variable exponents where the nonlinearity is modelled after the well known *p*-Laplace operator. Most of the theory untill now was different for the singular case and the degenerate case. While this was not problematic for constant exponent operators, when dealing with variable exponents, this led to an incomplete theory. In this paper, we develop an approach that can handle both the singular and degenerate cases simultaneously and as a consequence extend some of the previous a priori estimates in existing literature in a unified way.

Computational Methods for Waves in Complex Media: Part I Organizers: Wei Cai and Thomas Hagstrom

Wave propagation problems are of great importance in science and engineering. They typically involve multiple spatial and temporal scales ranging from propagation distances to the wavelengths to small geometric features and thus pose significant computational challenges. In recent years a number of effective methods have been developed for the basic equations of wave theory. In this minisymposium speakers will address the problems associated with generalizing these effective approaches to more complex problems.

Fast kernel matrix compression techniques for wave scattering in inhomogeneuous media Wei Cai, Southern Methodist University

We will present a hierarchical random compression method (HRCM) for kernel matrices in fast kernel summations. The HRCM combines the hierarchical framework of the H-matrix and a randomized sampling technique of column and row spaces for far-field interaction kernel matrices. We show that a uniform column/row sampling of a far-field kernel matrix, thus without the need and associated cost to pre-compute a costly sampling distribution, will give a low-rank compression of such low-rank matrices, independent of the matrix sizes and only dependent on the separation of the source and target locations. This far-field random compression technique is then implemented at each level of the hierarchical decomposition for general kernel matrices, resulting in an O(N logN) random compression method. Error and complexity analysis for the HRCM are included. Numerical results for electrostatic and Helmholtz wave kernels have validated the efficiency and accuracy of the proposed method with a cross-over matrix size, in comparison of direct $O(N^2)$ summations, in the order of thousands for a 3-4 digits relative accuracy for electrostatic and low frequency wave interaction kernels.

fast direct solver for scattering problems in quasi-periodic layered medium Adrianna Gillman, Rice University

Being able to accurately and efficiently solve quasi-periodic scattering problems involving multi-layered medium is important for the design of composite materials such as solar cells, photonic crystals and dielectric gratings for highpowered laser. Recently, Cho and Barnett proposed an integral formulation for these problems that is robust even at Wood's anomalies. In this talk, we present a fast direct solver for the linear system that results from discretizing that integral formulation. The direct solver scales linearly with respect to the number of discretization points and the precomputation is able to be reused for all right hand sides.

High-Order Semi-implicit IMEX WENO Scheme for the Euler System with All-Mach Number Jingmei Qiu, University of Delaware

We propose a high order asymptotic preserving method for all-Mach number simulations. In particular, we focus on finite difference schemes with weighted essentially non-oscillatory (WENO) reconstructions coupled with proper implicit-explicit (IMEX) Runge Kutta (RK) treatments for the system. The proposed method enjoys the following properties:

- 1. the schemes can robustly capture shock fronts in the compressible regime when the Mach number is of order 1;
- 2. the schemes automatically become high order, stable and consistent solvers for the incompressible Euler system when the Mach number approaches 0;
- 3. the schemes are high order accurate in both space and in time both when the acoustic waves are well-resolved and are under-resolved.

1D and 2D numerical results will be presented to showcase the method.

Efficient numerical simulation of spherical cloaking in time domain

Bo Wang, Southern Methodist University

We present a new model for the simulation of time domain electromagnetic cloaking. The new model only involves the electric displacement. Jump conditions across the interfaces between cloaking layer and other areas are derived from standard transmission conditions. The application of this new model in the simulation of spherical cloaking is studied. Due to the good features of the new model and the special geometry of the cloak, we introduce a highly efficient VSH-spectral-element discretization. The combination of the high order spectral element solver and NRBC in the discretization makes high accuracy. Numerical examples are given to validate the model and the efficiency and accuracy of the VSH-spectral-element discretization.

Computational Methods for Waves in Complex Media: Part II Organizers: Wei Cai and Thomas Hagstrom

Wave propagation problems are of great importance in science and engineering. They typically involve multiple spatial and temporal scales ranging from propagation distances to the wavelengths to small geometric features and thus pose significant computational challenges. In recent years a number of effective methods have been developed for the basic equations of wave theory. In this minisymposium speakers will address the problems associated with generalizing these effective approaches to more complex problems.

Bernstein-Bezier weight-adjusted discontinuous Galerkin methods for wave propagation in heterogeneous media

Jesse Chan, Rice University

High-order time-domain discontinuous Galerkin methods yield accurate numerical models of acoustic and elastic wave propagation on complex geometries due to their low numerical dispersion and dissipation. Additionally, the computationally intensive nature of time-explicit nodal discontinuous Galerkin methods is well-suited to implementation on Graphics Processing Units (GPUs). However, the computational runtime of DG methods increases rapidly as the order of approximation is increased. We discuss how to reduce these runtimes using properties of Bernstein polynomial bases, and describe how to leverage this structure in the presence of heterogeneous media with sub-element variations.

Iterative solvers for Discontinuous Galerkin methods

Minh Binh Tran, Southern Methodist University

We present a scalable iterative solver for high-order hybridized discontinuous Galerkin (HDG) discretizations of linear partial differential equations. It is an interplay between domain decomposition methods and HDG discretizations, and hence inheriting advances from both sides. In particular, the method can be viewed as a Gauss-Seidel approach that requires only independent element-by-element and face-by-face local solves in each iteration. As such, it is well-suited for current and future computing systems with massive concurrencies. Unlike conventional Gauss-Seidel schemes which are purely algebraic, the convergence of iHDG, thanks to the built-in HDG numerical flux, does not depend on the ordering of unknowns.

Multi-scale S-fraction reduced-order models for massive wavefield simulations Alexander V. Mamonov, University of Houston

We present a novel multi-scale method for large time-domain wavefield simulations. The algorithm consists of two stages. During the first off-line stage the computational domain is split into multiple subdomains. Then projectiontype reduced order models (ROMs) are computed for the partitioned operators at each subdomain. The off-line stage is embarrassingly parallel as ROM computations for the subdomains are independent of each other. At the second on-line stage the time-domain simulation is performed within the ROM framework. The crucial feature of our approach is the representation of the ROMs in terms of matrix Stieltjes continued fractions (S-fractions). This allows to sparsify the subdomain ROMs and thus reduce both the computation and communication costs. The performance of the method is illustrated on 3D composite anisotropic elastic problems.

Microdynamics in Regularized Brinkman Flow John Lagrone, Tulane University

The viscous flow of heterogeneous suspensions of small particles and fibers plays an important role in many biological, physical, and industrial processes. In order to better understand these processes, we investigate the dynamics of three dimensional elastic fibers in a shear flow within a porous medium. The medium is modeled using the Brinkman equations and we aim to characterize the effect of varying the Brinkman parameter, which represents resistance due to obstacles in the flow, on deformations of the fiber, buckling dynamics, and periodic orbits. The fluid interactions are modeled using regularized fundamental solutions to the Brinkman equations coupled to the elastic forces in the fiber, which is constructed from a network of springs.

Recent advances in modeling, simulation and stability studies in EOR and geosciences Organizers: Prabir Daripa, Department of Mathematics, TAMU

This mini-symposium brings together a collection of pressing challenges and some recent advances in mathematical modeling, analysis of associated PDEs, computation, and stability studies in porous media flows motivated by applications in enhanced oil recovery and geosciences.

Mathematical and computational challenges for multi-phase porous media flows in EOR Prabir Daripa, Department of Mathematics, TAMU

We will first give an overview of recent progress made by our team in the modeling, simulation and stability studies of multi-component multi-phase fluid flow. Then we will discuss some current ongoing work in this area motivated by a need to develop proper modeling and simulation techniques for displacement processes involving one or more non-Newtonian fluids. There are challenges at various levels including stability studies of such flows which we will discuss in this talk. This work has been possible due to financial support from the U.S. National Science Foundation grant DMS-1522782. Part of this talk may be based on my current ongoing joint work with Zhiying Hai and past work with Sourav Dutta and Craig Gin.

A study on dispersion in multicomponent porous media flow and transport processes Sourav Dutta, US Army Engineer Research & Development Center, Mississippi

A hybrid numerical method is used to solve a global pressure based porous media flow model of chemical enhanced oil recovery [1,2]. The global pressure formulation incorporates the effect of multiple components and capillary pressure between the two fluid phases. The numerical method is based on the combination of a discontinuous finite element method and the modified method of characteristics. The impact of molecular diffusion, mechanical dispersion, various injection policies and chemical properties on the evolution of scalar concentration distributions are studied through numerical simulations of various flooding schemes. The relative importance of the advective, capillary diffusive and dispersive fluxes are compared over different flow regimes defined in the parameter space of Capillary number, Peclet number, longitudinal and transverse dispersion coefficients. Such studies are relevant for the design of effective injection policies and determining optimal combinations of chemical components for improving recovery. This work has been possible due to financial support from the U.S. National Science Foundation grant DMS-1522782. The talk is based on joint work with Prabir Daripa.

[1] P. Daripa, S. Dutta, Modeling and simulation of surfactant-polymer flooding using a new hybrid method, Journal of Computational Physics **335**, 249–282 (2017).

[2] P. Daripa, S. Dutta, Convergence analysis of characteristics-based hybrid method for multicomponent transport in porous media, arXiv:1707.00035 [math.NA] (2017).

Linear stability of multi-layer radial porous media and Hele-Shaw flows

Craig Gin, Department of Applied Mathematics, University of Washington, Seattle

The viscous fingering instability is an important limiting factor in oil recovery. Thus, it was oil recovery that led Saffman and Taylor to produce their seminal work on the stability of Hele-Shaw flows in order to understand water flooding. In order to design efficient flooding schemes of chemical enhanced oil recovery, we have been investigating this instability for multiphase multi-component immiscible Newtonian fluid flows with more than two fluid regions in rectilinear and radial geometries [1,2,3]. We have also been investigating dispersive effect of species on the maximum instability [4]. In this talk, we present some recent results on the stability of multi-layer radial Hele-Shaw flows of Newtonian fluids. In particular, we consider time-dependent injection strategies and how they are affected by increasing the number fluid layers [5]. We also look at the stability of flows that contain a fluid with variable viscosity [6]. This work has been possible due to financial support from the U.S. National Science Foundation grant DMS-1522782.

The talk is based on joint work with Prabir Daripa.

[1] P. Daripa, Hydrodynamic Stability of Multi-Layer Hele-Shaw Flows, J. Stat. Mech. 2008 (2008) 32 pages, Article No. P12005.

[2] P. Daripa, Studies on Stability in Three-Layer Hele-Shaw Flows, Phys. Fluids, **20**, Issue 11 (2008) Article No. 112101.

[3] C. Gin and P. Daripa, Stability Results for Multi-Layer Radial Hele-Shaw and Porous Media Flows, Phys. of Fluids, 27, 012101 2014.

[4] P. Daripa and Craig Gin, Studies on Dispersive Stabilization of Porous Media Flows, Phys. Fluids, 28, 082105, 2016.

[5] C. Gin and P. Daripa, Time-dependent injection strategies and interfacial stability in multi-layer Hele-Shaw and porous media flows, Under Review.

[6] C. Gin and P. Daripa, Stability Results on Radial Porous Media and Hele-Shaw Flows with Variable Viscosity between Two Moving Interfaces, Under Review.

Pore-Scale Analysis of Interface Instabilities and Determination of Effective Permeability using Lattice Boltzmann Method

Mayank Tyagi, Craft & Hawkins Department of Petroleum Engineering, LSU

Fluid displacement in porous media is an important class of flow to understand enhanced oil recovery mechanisms in complex environments. Immiscible fluid interface instabilities during flow through porous media occur often and the prediction of water and oil flow pattern is essential to determine the efficiency of oil recovery processes. Although numerous experimental and numerical studies have been conducted in the past to reproduce and analyze the interfacial instabilities during immiscible fluid displacement in porous media, only a few of such studies show the flow details at complex 3-D pore-scale and relate the mechanisms to macroscale information. In order to better understand the interface instabilities phenomenon under different force balance conditions, several pore-scale Lattice Boltzmann Method (LBM) simulations are performed.

In this study, a multiphase Lattice Boltzmann Method is used to simulate two different types of interface instabilities caused by the relative imbalance between viscous- and capillary- forces in the pore spaces of sphere packs. Features of interface instabilities controlled due to this imbalance between viscous forces and capillary forces is qualitatively demonstrated as the sharp interface appears in the pore space when the flow mechanism is viscosity dominated and along the pore-grain surface when the flow mechanism is capillary dominated. Further, the flow paths of two different cases are different even in the same porous medium geometry. In addition, the effective relative permeability is determined for heterogeneous porous media. Based upon LBM simulations of fluid displacement in sphere packs, it is observed that the pore-scale heterogeneity will affect relative permeability of non-wetting phase much more than the wetting phase relative permeability values.

The talk is based on joint work with Zhipeng Zhu at LSU

Nonlinear modeling of disease dynamics

Organizers: Kun Gou, Md Rafiul Islam, Tamer Oraby

Researchers are going to present their recent work on modeling diseases for living bodies. The presentations demonstrate new ideas about disease initiation, its spread, or control. The diseases are either infectious or non-infectious, and deterministic or stochastic approaches are applied in the analysis. This event connects researchers or students with the same interest for further exploration of similar topics.

Modeling an outbreak of the Middle East Respiratory Syndrome in a hospital

Tamer Oraby(1), Yasar Tasnif(2), Mustafa Al-Zoughool(3), Adriana Quiroz(1), Zeinab Mohamed(1), Hanan Balkhy(3), 1: School of Mathematical and Statistical Sciences, University of Texas Rio Grande Valley, Edinburg, Texas, USA. 2: College of Health Affairs, Cooperative Pharmacy Program, University of Texas Rio Grande Valley, Edinburg, Texas, USA. 3: Department of Community and Environmental Health, College of Public Health and Health Informatics, King Saud Bin Abdulaziz University for Health Sciences, Riyadh, Saudi Arabia.

Middle East Respiratory Syndrome-coronavirus (MERS-CoV), a virus with a high fatality rate, is spreading in the Middle East, especially in Saudi Arabia (SA) which is its point of origin. MERS-CoVs nosocomial outbreaks are common in SA. In a collaboration with researchers and health officials in Saudi Arabia we are constructing a model to depict the MERS-CoV nosocomial outbreak in a SA hospital. We are using the model to estimate different parameters related to the outbreak and to test the effect of the infectious disease control plan in that hospital. This model will assist to derive lessons from this particular nosocomial outbreak to inform and update infection control policy. In this talk, I am going to present the latest results in that project.

Mathematical modeling of an infectious disease and identification of its dominant transmission pathways

Md Rafiul Islam(a), Patrick J. Cusaac(b), Matthew J. Gray(b), Angela Peace(a), a: Texas Tech University, Lubbock, TX, b: University of Tennessee Institute of Agriculture, Knoxville, TN

We developed and analyzed a Susceptible-Infected-Recovered-Susceptible (SIRS) type disease models for emerging fungal pathogen Batrachochytrium salamandrivorans (Bsal). Our models included two routes of pathogen transmission: direct transmission via contact between infected and susceptible individuals and environmental transmission via shed zoospores in the water. Unlike previous models, we categorized individuals into multiple stages of infection. We found the invasion probability for Bsal (i.e., the basic reproductive number, \mathcal{R}_0) into a population of the Eastern Newt adults. We performed numerical sumulatons and parameter sensitivity analysis using Latin hypercube sampling and partial rank coefficient correlation. We identified dominant transmission pathways and discussed the intervention strategies.

Ebola: Impact of hospital's admission policy in an overwhelmed scenario

Mondal Hasan Zahid, Christopher M. Kribs, Department of Mathematics, University of Texas at Arlington, Arlington, TX

Infectious disease outbreaks sometimes overwhelm healthcare facilities. A recent case occurred in West Africa in 2014 when an Ebola virus outbreak overwhelmed facilities in Sierra Leone, Guinea and Liberia. In such scenarios, how many patients can hospitals admit to minimize disease burden? This study considers what type of hospital admission policy during a hypothetical Ebola outbreak can better serve the community, if overcrowding degrades the hospital setting. Our result shows that which policy minimizes loss to the community depends on the initial estimation of the control reproduction number, R_0 . When the outbreak grows extremely fast ($R_0 >> 1$) it is better (in terms of total disease burden) to stop admitting patients after reaching the carrying capacity because overcrowding in the hospital

makes the hospital setting ineffective at containing infection, but when the outbreak grows only a little faster than the systems ability to contain it $(R_0 \gtrsim 1)$, it is better to admit patients beyond the carrying capacity because limited overcrowding still reduces infection more in the community. However, when R_0 is no more than a little greater than 1 (for our parameter values, 1.012), both policies result the same because the number of patients never exceeds the maximum capacity.

Nonlinear Tubular Organ Deformation Analysis for Airway Swelling

Kun Gou(1), Thomas J Pence(2), Pak-Wing Fok(3), Yibin Fu(4), 1: Department of Science and Mathematics, Texas A&M University-San Antonio, San Antonio, TX. 2: Department of Mechanical Engineering, Michigan State University, East Lansing, MI. 3: Department of Mathematical Sciences, University of Delaware, Newark, Delaware. 4: School of Computing and Mathematics, Keele University, Staffordshire, ST5 5BG, UK

We study one of the important human tubular organs, the trachea, under deformation caused by the disease angioedema. This pathology can suddenly increase the volume of the trachea, and cause serious breathing difficulty. Nonlinear behaviors of the tubular radius change are illustrated to show how the trachea luminal size alteration depends on the swelling parameters and their effect on modifying tissue stiffness. The possibility of complete tracheal channel closure is also studied to understand if it is possible for the angioedema to close the airway. This study serves as an exemplary study on nonlinear deformation behaviors of human tubular organs with multiple layers.

Nonlinear Conservation Laws and Applications, Part I Organizers: Kun Zhao and Yanni Zeng

Hyperbolic conservation laws, a mathematical field with a rich history, is a topic which has received attention for both its development of mathematical analysis as well as the important applications the analysis addresses. We aim to bring together leading experts in the field as well as younger research mathematicians to present the significant theoretical advances and discuss applications. The theory has been developed in conjunction with applications in gas dynamics, fluid dynamics, magneto-hydrodynamics, nonlinear elasticity, combustion models and mathematical biology. Applications arising in the field of hyperbolic conservation laws and related areas include traffic models, large scale supply-chain modeling, the kinetic theory of gases, plasma physics, the mathematical theory of semiconductors, general relativity, cancer modeling and blood flow models.

On Similarity Flows For the Compressible Euler System

Charis Tsikkou, University of West Virginia

We review the construction of globally defined radial similarity shock and cavity flows, and give a detailed description of their behavior following collapse. We then prove that similarity shock solutions provide bona fide weak solutions, of unbounded amplitude, to the multi-dimensional Euler system.

We also point out that both types of similarity flows involve regions of vanishing pressure prior to collapse (due to vanishing temperature and vacuum, respectively) - raising the possibility that Euler flows may remain bounded in the absence of such regions. This is joint work with Helge Kristian Jenssen (PSU).

Recent Results on Kolmogorov Entropy Compactness Estimates for Conservation Laws Tien Khai Nguyen, North Carolina State University

Inspired by a question posed by Lax in 2002, in recent years it has received an increasing attention the study on the quantitative analysis of compactness for nonlinear PDEs. In this talk, I will present recent results on the sharp compactness estimates—in terms of Kolmogorov ε -entropy in \mathbf{L}^{1} - for hyperbolic conservation laws. Estimates of this type play a central roles in various areas of information theory and statistics as well as of ergodic and learning theory. In the present setting, this concept could provide a measure of the order of "resolution" of a numerical method for the corresponding equation.

Global Regularity for Burgers Equation with Density Dependent Fractional Dissipation Changhui Tan, University of South Carolina

Fractional Burgers equations are a family of equations which connect inviscid and viscous Burgers equations. It is well-known that if the dissipation is strong, the solution is globally regular. On the other hand, it the dissipation is weak (called supercritical case), the solution can lose regularity at a finite time. In this talk, I will introduce a model where the dissipation depends on density. The model is motivated by self-organized dynamics in math biology. Despite that the equation shares a lot of similarities to fractional Burgers equation, the solution is globally regular, even in the supercritical case. I will explain the regularization mechanism that is due to the nonlocal nonlinear modulation of dissipation.

Compensated Compactness, (Isometric) Immersions and Elasticity Siran Li, Rice University and McGill University

We survey some recent results on the compactness of immersions of Riemannian and semi-Riemannian manifolds. The main ingredient involved is the method of compensated compactness, which is key to the study of hyperbolic conservation laws. For our purpose, a geometric compensated compactness framework is developed, utilising techniques from harmonic and geometric analysis. We shall also discuss the applications to the stability results in non-linear elasticity. Joint work with Prof. Gui-Qiang G. Chen (Oxford, UK) and Marshall Slemrod (Wisconsin-Madison, USA and Weizmann Institute, Isreal).

Nonlinear Conservation Laws and Applications, Part II Organizers: Kun Zhao and Yanni Zeng

Hyperbolic conservation laws, a mathematical field with a rich history, is a topic which has received attention for both its development of mathematical analysis as well as the important applications the analysis addresses. We aim to bring together leading experts in the field as well as younger research mathematicians to present the significant theoretical advances and discuss applications. The theory has been developed in conjunction with applications in gas dynamics, fluid dynamics, magneto-hydrodynamics, nonlinear elasticity, combustion models and mathematical biology. Applications arising in the field of hyperbolic conservation laws and related areas include traffic models, large scale supply-chain modeling, the kinetic theory of gases, plasma physics, the mathematical theory of semiconductors, general relativity, cancer modeling and blood flow models.

Recent Results for the Logarithmic Keller-Segel-Fisher/KPP System

Yanni Zeng, University of Alabama - Birmingham

We consider a Keller-Segel type chemotaxis model with logarithmic sensitivity and logistic growth. It is a 2×2 system describing the interaction of cells and a chemical signal. We study Cauchy problem with finite initial data, i.e., without the commonly used smallness assumption on initial perturbations around a constant ground state. We survey a sequence of recent results by the authors on the existence of global-in-time solution, long-time behavior, vanishing coefficient limit and optimal time decay rates of the solution.

BV Existence or Blowup for p-system?

Geng Chen, University of Kansas

In this talk, we will discuss the recent front tracking example found by Bressan, Zhang and I, which shows the possible BV blowup pattern for the isentropic Euler equations (p-system). A very interesting question is whether such a pattern can exist exactly or not. We will also discuss the recent progress on the BV estimate for large solutions of p-system.

(-1)-homogeneous Solutions of Stationary Incompressible Navier-Stokes Equations with Singular Rays

Xukai Yan, Georgia Institute of Technology

In 1944, L.D. Landau first discovered explicit (-1)-homogeneous solutions of 3-d stationary incompressible Navier-Stokes equations (NSE) with precisely one singularity at the origin, which are axisymmetric with no swirl. These solutions are now called Landau solutions. In 1998 G. Tian and Z. Xin proved that all solutions which are (-1) homogeneous, axisymmetric with one singularity are Landau solutions. In 2006 V. Sverak proved that with just the (-1)-homogeneous assumption Landau solutions are the only solutions with one singularity. Our work focuses on the (-1)-homogeneous solutions of 3-d incompressible stationary NSE with finitely many singularities on the unit sphere. In this talk we will first classify all (-1)-homogeneous axisymmetric no-swirl solutions of 3-d stationary incompressible NSE with one singularity at the south pole on the unit sphere as a two dimensional solution surface. We will then present our results on the existence of a one parameter family of (-1)-homogeneous axisymmetric solutions with non-zero swirl and smooth on the unit sphere away from the south pole, emanating from the two dimensional surface of axisymmetric no-swirl solutions. We will also present asymptotic behavior of general (-1)-homogeneous axisymmetric solutions in a cone containing the south pole with a singularity at the south poles.

This is a joint work with Professor Yanyan Li and Li Li.

A New Approach for Designing Moving-Water Equilibria Preserving Schemes for the Shallow Water Equation

Tong Wu, Tulane University

Shallow water models are widely used to describe and study free-surface water flow. They are hyperbolic systems of balance law and usually solved by finite volume methods, which are appropriate numerical tools for computing nonsmooth solutions. One requirement when designing numerical schemes for shallow water models is to preserve a delicate balance between the flux and source terms since many physical related solutions are small perturbations of some steady state solutions. In this presentation, we construct a new general approach of designing well-balanced central-upwind schemes for shallow water models, which can capture and preserve the "lake at rest" steady states and also the moving steady states, even with complicated bottom frictions, and illustrate their performance on a number of numerical examples. This is joint work with Yuanzhen Cheng, Alina Chertock, Michael Herty, and Alexander Kurganov.

Switched Systems in Controls

Organizers: Oleg Makarenkov

Stability and Robustness Analysis for Switched Systems with Time-Varying Delays Michael Malisoff, Louisiana State University

A new technique is presented for the stability and robustness analysis of nonlinear switched time-varying systems with uncertainties and time-varying delays. The delays are allowed to be discontinuous (but are required to be piecewise continuous) and arbitrarily long with known upper bounds. The technique uses an adaptation of Halanay's inequality and a trajectory-based technique, and is used for designing switched controllers to stabilize linear time-varying systems with time-varying delays.

State Consensus for Discrete-time Multi-agent Systems over Time-varying Graphs Guoxiang Gu, Louisiana State University

We study the state consensus problem for linear shift-invariant discrete-time homogeneous multi-agent systems (MASs) over time-varying graphs. A novel approach based on the small gain theorem is proposed to design the consensus control protocols for both neutrally stable and neutrally unstable MASs, assuming the uniformly connected graphs. It is shown that the state consensus can be achieved for neutrally stable MASs under a weak uniform observability condition; for neutrally unstable MASs, the state consensus entails a strong uniform observability condition. Two numerical examples are worked out to illustrate our consensus results.

Formation Control in Multi-Robot Systems

Kaveh Fathian, University of Texas at Dallas

Thanks to recent technological advances, it is now possible to deploy a large number of robots to collaboratively map an environment, inspect infrastructure, or deliver goods. Given a swarm of robots, the question remains: How can robots safely perform a collaborative task by making their own decisions based on local perception of their environment? We approach this question by studying the distributed formation control problem, in which a desired geometric arrangement of robots emerges from their individual behavior. This is a fundamental building block upon which more sophisticated tasks can be constructed. We show how this problem can be modeled mathematically and how novel applications of mathematical machinery in algebraic graph theory, control and dynamical systems, and switched systems theory can be used to find a solution. Lastly, we discuss open problems and future avenues of research.

Convex Analysis Approach to Stabilize Spring Systems with Switching Topology Oleg Makarenkov, University of Texas at Dallas

In our earlier work (https://arxiv.org/abs/1708.03084) we applied the Moreau sweeping process framework to model the evolution of stresses in networks of elastoplastic springs under the action of periodic loading. I will briefly discuss this achievement and then show how this framework can be adapted to the case where the topology of the network changes in time. A particular situation that will be addressed is when reaching the yield surface causes breakage of a spring and its reappearing at a different location, which takes place in self-healing materials.

Models and methods in biology and physics: From the stochastic to the continuum, Part I Organizers: William Ott, University of Houston

Biophysical phenomena may be modeled on multiple spatiotemporal scales. The stochastic and continuum points of view, long central within theoretical and applied physics, now permeate the modeling of biological processes. This minisymposium aims to present modern theoretical and computational techniques, as well as timely advances in modeling and model analysis. Topics include synthetic biocircuit design and analysis, spatial Moran modeling of pattern formation, neuronal activity in the hippocampus, eco-evolutionary feedback and implications for species coexistence, stochastic mode reduction for geophysical flows, and non-parametric estimation of drift and diffusion for stochastic differential equations.

Impact of Delay on the Dynamics of Biophysical Systems William Ott, University of Houston

Order one time delay appears in biophysical systems at several scales. In the context of genetic regulatory circuits, for instance, delay results from transcription, translation, and the post-translational processes required to produce functional regulator protein. At the systems biology scale, for instance, delay inescapably impacts the dynamics of the glucose-insulin system in humans. Such forms of biophysical delay, themselves random, interact with other stochastic aspects of the dynamics, thereby producing interesting phenomena and rich mathematical questions. In this talk, we survey recent results on identifying and analyzing delay-induced phenomena.

Continuum Modeling of Bacterial Growth in Confined Environments: Stochastic Interactions and Nematic Ordering

James Winkle, Rice University

Bacteria in natural environments are prevalent in close-packed biofilm structures, where intercellular chemical and physical signaling interactions can lead to emergent forms. In experimental synthetic biology, close-packing of cells also exists in microfluidic devices that study bacterial consortia in a continuous, exponential growth phase. Due to their anisotropic, rod-like geometry and axial growth, bacteria in microfluidic experiments exhibit nematic ordering dynamics similar to those of liquid crystals under shear flow. The relative rate of nematic ordering can be vastly different, however, depending on the geometry of the device and cell location with respect to trap boundaries. We extend a continuum model of bacterial growth and nematic ordering to capture the 2D spatiotemporal dynamics of a monolayer of bacterial cells grown in a microfluidic trap. Typical bacterial growth environments of low Reynolds number result in a Poisson's equation PDE formulation for cell growth pressure, which is coupled to a dynamical equation for the tensor order parameter under shear flow. We numerically simulate the growth and ordering dynamics and demonstrate how stochastic interactions between cells can lead to persistent destabilization of cell ordering in regions predicted by the model.

Stochastic Subgrid-Scale Parameterization for Spatial Averages in One-Dimensional Shallow Water Dynamics

Ilya Timofeyev, University of Houston

We address the question of parameterizing the subgrid scales in simulations of geophysical flows by applying stochastic mode reduction to the one-dimensional stochastically forced shallow water equations. The problem is formulated in physical space by defining resolved variables as local spatial averages over finite-volume cells and unresolved variables as corresponding residuals. Based on the assumption of a time-scale separation between the slow spatial averages and the fast residuals, the stochastic mode reduction procedure is used to obtain a low-resolution model for the spatial averages alone with local stochastic subgrid-scale parameterization coupling each resolved variable only to a few neighboring cells. The closure improves the results of the lowresolution model and outperforms two purely empirical stochastic parameterizations. It is shown that the largest benefit is in the representation of the energy spectrum. By adjusting only a single coefficient (the strength of the noise) we observe that there is a potential for improving the performance of the parameterization, if additional tuning of the coefficients is performed. In addition, the scale-awareness of the parameterizations is studied.

Cell Assembly Detection and Low Dimensional Dynamics Extraction for Hippocampal Calcium Imaging Data

Duc Truong, Southern Methodist University

Calcium imaging is capable of recording simultaneous activity of hundreds of neurons, resulting in complicated and high dimensional data sets. In the hippocampus, neuronal activity is known to be associated with encoding and retrieving contextual memory, but the precise meaning of this activity is not clear. We apply novel theoretical tools to imaging data recorded during fear conditioning in two strains of mice that show different levels of ability to generalize memories.

First, we apply a new algorithm to detect cell assemblies, which are groups of cells working together and being proven to associate with cognitive tasks. We show that algorithm can be applied to wider class of data sets than anticipated by the author. The result shows that cells participating in assemblies are more spatially coherent, and that the spatial coherences are different between two strains.

Second, we apply Poisson linear dynamic system (PLDS) model with external input to construct a low dimensional representation of neuron populations activity. The preliminary approximated parameters from the model also distinguished two strains.

Models and methods in biology and physics: From the stochastic to the continuum, Part II Organizers: William Ott, University of Houston

Biophysical phenomena may be modeled on multiple spatiotemporal scales. The stochastic and continuum points of view, long central within theoretical and applied physics, now permeate the modeling of biological processes. This minisymposium aims to present modern theoretical and computational techniques, as well as timely advances in modeling and model analysis. Topics include synthetic biocircuit design and analysis, spatial Moran modeling of pattern formation, neuronal activity in the hippocampus, eco-evolutionary feedback and implications for species coexistence, stochastic mode reduction for geophysical flows, and non-parametric estimation of drift and diffusion for stochastic differential equations.

Feedbacks between Ecology and Evolution Affect Coexistence of Different Species Swati Patel, Tulane University

Ecological communities typically consist of many different interacting species. One central goal of ecology is to understand the mechanisms that allow these species to coexist with one another. Traditionally, models describing communities ignored evolution of the species. However recent evidence suggests that evolution within a species and feedbacks between ecology and evolution can have important consequences on the community. In this talk, I discuss how these eco-evolutionary feedbacks affect the coexistence of species. In particular, I use the theories of permanence and singular perturbations to give results on the persistence and stability of populations, in light of eco-evolutionary feedbacks.

Moran Models of Spatial Alignment in Microbial Colonies

Bhargav Karamched, University of Houston and Rice University

We describe a spatial Moran model that captures mechanical interactions and directional growth in spatially extended populations. The model is analytically tractable and completely solvable under a mean-field approximation, and can elucidate the mechanisms that drive the formation of population-level patterns. As an example we model a population of $E.\ coli$ growing in a rectangular microfluidic trap. We show that spatial patterns can arise as a result of a tug-of-war between boundary effects and growth rate modulations due to cell-cell interactions: Cells align parallel to the long side of the trap when boundary effects dominate. However, when cell-cell interactions exceed a critical value, cells align orthogonally to the trap's long side. This modeling approach and analysis can be extended to directionally-growing cells in a variety of domains to provide insight into how local and global interactions shape collective behavior.

Bayesian Parameter Inference for Models of Microbial Consortia Mehdi Sadeghpour, University of Houston and Rice University

We provide models for microbial consortia (a combination of two or more microbial colonies growing in the same environment) and based on these models and experimental data, we estimate the values of the parameters of interest in the model together with an analysis of uncertainty levels in the parameter estimation. In particular, we study quorum sensing (signaling between bacteria) in microbial consortia with different connectivity network structures. Bacterial signaling is enabled by bacteria sending diffusive molecules out of the cell and into other neighboring cells. The models we use are deterministic models consisting of ordinary differential equations. Parameter inference algorithms are developed based on Bayesian parameter estimation approaches where the amount of experimental data is limited. In the core of the Bayesian parameter estimation approaches, we use Markov Chain Monte Carlo (MCMC) sampling methods, such as Metropolis sampling. We have created a mechanistic model for a microbial consortium that consists of 3 strains signaling to each other and we can infer the parameters such as the cell growth rates and protein synthesis rates. Our experimental observations consist of cell population (optical density) data and protein fluorescence data in (12-18)-hour long experiments. The two main future goals of this research are: finding robust mechanistic models that can be useful for a variety of different microbial consortia for prediction and design purposes, and presenting an effective parameter estimation approach that can infer the parameter values efficiently from, often limited, experimental observations.

Consistency of Non-Parameteric Estimation of Drift and Diffusion Coefficients in SDEs from Stationary Time-Series

Xi Chen, University of Houston

We study non-parametric estimation of drift and diffusion coefficients in stochastic differential equations. Three parameters affect the accuracy of non-parametric method, namely the observational time-step, Δt , number of observed data-points, M, space-discretization step for the drift and diffusion functions, Δx . We derive optimal sub-sampling criteria for obtaining consistency of non-parametric estimation and balancing errors arising from the three parameters discussed above. We present several numerical examples and practical reciepe for selecting optimal values of these three parameters.

Spectral Theory of Differential Operators, Part I Organizers: Stephen P. Shipman

This pair of mini-symposia bring together a collection of advances in the multifarious realm of spectral theory of differential operators. The presentations have the flavor classical problems in modern incarnations.

Nodal count distribution of graph eigenfunctions Gregory Berkolaiko, TX A&M

We start by reviewing the notion of "quantum graph", its eigenfunctions and the problem of counting the number of their zeros. The nodal surplus of the *n*-th eigenfunction is defined as the number of its zeros minus (n - 1). When the graph is composed of two or more blocks separated by bridges, we propose a way to define a "local nodal surplus" of a given block. Since the eigenfunction index n has no local meaning, the local nodal surplus has to be defined in an indirect way via the nodal-magnetic theorem of Berkolaiko, Colin de Verdiére and Weyand.

We will discuss the properties of the local nodal surplus and their consequences. In particular, its symmetry properties allow us to prove the long-standing conjecture that the nodal surplus distribution for graphs with ? disjoint loops is binomial with parameters (β , 1/2).

The talk is based on joint work with Lior Alon and Ram Band, arXiv:1709.10413 (accepted to CMP).

On Liouville-Riemann-Roch theorems

Minh Kha, University of Arizona

A generalization by Nadirashvili, and then Gromov and Shubin of the classical Riemann-Roch theorem describes the index of an elliptic operator on a compact manifold with a divisor of prescribed zeros and allowed singularities. On the other hand, Liouville type theorems count the number of solutions of a given polynomial growth of the Laplace-Beltrami (or more general elliptic) equation on a non-compact manifold. The solution of a 1975 Yau's conjecture by Colding and Minicozzi implies in particular, that such dimensions are finite for Laplace-Beltrami equation on a nilpotent co-compact covering. In the case of an abelian covering, much more complete Liouville theorems (including exact formulas for dimensions) have been obtained by Kuchment and Pinchover. One wonders whether such results have a combined generalization that would allow for a divisor that "includes the infinity." Surprisingly, combining the two types of results turns out being non-trivial. The talk will present such a result obtained recently in a joint work with Peter Kuchment.

Quotients of finite-dimensional operators by symmetry representations

Wen Liu, Lamar University, Beaumont, TX

A finite-dimensional operator which commutes with some symmetry group admits quotient operators, determined by the group action and a choice of a representation of this group. Taking the quotient isolates the part of the spectrum supporting the chosen representation, reduces complexity of the problem. We develop a complete and computationally convenient set of tools to extract the quotient of an operator by a representation of its symmetry group", or quotient operator for short, in the context of finite dimensional operators. Specifically, we incorporate new insights to explicitly construct the (generalized) blocks of the quotient operators, to catalog their algebraic and spectral properties and thus to bring the current notions of quotient systems into the discrete setting. We will present an elementary example in order to demonstrate the ideas behind the quotient operator and the possible ways to construct it. Joint work with Gregory Berkolaiko, Ram Band, and Chris Joyner.

Spectral Analysis in Renormalization Stephen Fulling, TAMU

Some recent applications of geometric asymptotics in quantum field theories subject to external conditions will be summarized.

Spectral Theory of Differential Operators, Part II Organizers: Stephen P. Shipman

This pair of mini-symposia bring together a collection of advances in the multifarious realm of spectral theory of differential operators. The presentations have the flavor classical problems in modern incarnations.

Quantitative bounds versus weakly coupled states for generalized Schrödinger Operators Vu Hoang, UT San Antonio

In this talk I consider various quantum mechanical operators having the form (kinetic energy)+(potential energy). The kinetic energy term is more general than usually assumed. I present a condition on the kinetic energy that guarantees the existence of bound states for arbitrarily weak attractive potentials. If the condition fails, weakly coupled bound states do not exist and we have a quantitative bound on the number of eigenvalues below the essential spectrum. Applications include pseudorelativistic one-body and two-body operators. (work with D. Hundertmark, J. Richter, S. Vugalter)

Birman-Hardy-Rellich-type Inequalities and Refinements

Isaac Michael, Baylor University

In 1961, Birman proved a sequence of inequalities for smooth functions of compact support on the open half-line, containing as special cases the classical Hardy and Rellich inequalities. In this talk, we show this sequence of inequalities holds on a more general Hilbert space of functions defined on the closed half-line. We also show that Birman's inequalities are closely related to a sequence of generalized, continuous Cesaro averaging operators whose spectral properties we determine.

Finally, we hint at generalizations in the multi-dimensional setting, focusing on improved inequalities with logarithmic and radial refinement terms. In particular, we derive Hardy-type multidimensional inequalities with logarithmic refinement terms, and the gradient replaced by its radial counterpart.

This is based on joint work with F. Gesztesy, L. Littlejohn, M. Pang, and R. Wellman.

Construction of Orthogonal Eigenfunction Bases of Hilbert-Sobolev Spaces Giles Auchmuty, University of Houston

Throughout science and engineering, the solutions of linear partial differential equations are approximated for analytical purposes by "eigenfunction expansions". These approximations are expected to be a finite number of terms of an exact solution where the eigenfunctions can be proved to be an orthogonal basis of a Hilbert space.

However some eigenfunction expansions have very poor convergence properties—and certainly dont converge pointwise even if they can be proven to converge in L^2 . This talk will describe some results about solving Laplace's equation on a rectangle in the plane subject to Dirichlet, Robin and Neumann conditions.

Non-convergent, poorly convergent and rapidly convergent eigenfunction expansions for the same problem will be described based on numerical computations by Manki Cho. The results may be interpreted by looking at whether the eigenfunctions actually are bases with respect to different inner products in relevant Hilbert-Sobolev spaces.

Analytic continuation of bilinear forms and spectra of divergence form operators for composite media.

Robert Lipton, LSU

We discuss analytic continuation techniques for extending divergence form operators originally defined for positive real valued coefficients to coefficients with complex values. Application of these techniques provide new methods for characterization of spectra of high contrast media with application to acoustics and design of photonic bandgaps and negative index metamaterials. This is joint work with Yue Chen and Robert Viator.

Modeling, analysis, and computation in mathematical biology Organizers: Xiang-Sheng Wang

Over the past decades, the area of mathematical biology has gained an increasing attention and received a growing momentum. In this mini-symposium, we will bring together researchers from various backgrounds to work together and contribute to the study of biological problems from the perspectives of modeling, analysis and computation. It will serve as a platform to present recent progresses, exchange research ideas, extend academic networks, and seek future cooperation. Speakers and talks are carefully selected to make the mini-symposium interesting and attractive to a diverse audience.

Optimal control of a continuously size-structured model for the growth and treatment of metastatic cancer

Jun Liu, Southern Illinois University Edwardsville

We propose a new unified continuously size-structured model for the growth of metastatic tumors, which extends a well-known dynamical model given in [K. Iwata, K. Kawasaki and N. Shigesada, A dynamical model for the growth and size distribution of multiple metastatic tumours. J. Theor. Biol., 203 (2000) 177-186.] The optimal treatment of metastatic cancer based on the proposed model is investigated with PDE optimal control theory, where both optimize-then-discretize and discretize-then- optimizae algorithms are considered for solving the proposed treatment model. Numerical simulations are provided to validate our algorithms and illustrate the biological meaning of the treatment outcomes.

Spatially heterogeneous producer-grazer model subject to stoichiometric constraints Md Masud Rana, Texas Tech University

The LKE model, one of the classic stoichiometric producer-grazer model developed by Loladze I, Kuang Y, Elser JJ (2000), was extended spatially by Dissanayake C (2016). The model in Dissanayake C (2016) tracks both, quantity and quality, of the producer population in space, assessing quality through the ratio of two essential elements phosphorus to carbon (P:C) of the producer. But both models neglect to track the free phosphorus in the media and assume all the phosphorus in the system is kept between the producer and the grazer. Wang H, Kuang Y, Loladze I (2008) eliminate this assumption to extend the LKE to a model that tracks phosphorus content in the producer and free phosphorus in the media, but as the LKE, it neglects spatial dynamics. Here we extend the spatially heterogeneous model of Dissanayake C (2016), to track the phosphorus content in the producer and free phosphorus in the media. We simulate our model numerically under various environmental conditions. In similar conditions to those in Dissanayake C (2016), we find cases where qualitatively different behavior can be observed when both, phosphorus in the producer and free phosphorus in the media, are tracked.

An adaptive MFS for The Laplace Equation in 2D and 3D

Jaeyoun Oh, The University of Southern Mississippi

We present an adaptive method of fundamental solutions (MFS) for solving the Laplace equation in 2D and 3D settings. An error estimator on the domain boundary is defined to refine the distributions of collocation points on the boundary of the domain and source points outside the domain. Numerical results show that the proposed adaptive MFS is more accurate and stable than using uniformly distributed source points, especially for non-harmonic or non-smooth boundary conditions.

A diffusive prey-predator model with Alee effect in predator

Zhifu Xie, The University of Southern Mississippi

We investigate whether hunting cooperation can promote Allee effects in a pre-predator model. Diffusion and cross diffusion are introduced into both prey and predator. In this talk, we present some preliminary results and some numerical simulations.

Mathematical and computational aspects of fracture, Part I

Organizers: Blaise Bourdin, (bourdin@lsu.edu) LSU, and Robert Lipton, (lipton@lsu.edu) LSU

Coupling methods of nonlocal and local models

Xiaochuan Tian, Mathematics Department University of Texas, Austin

Multiscale models for materials with fractures or defects involve local interaction where classical models work well and nonlocal interaction where defects display. In this talk, we present two types of energy-based coupling methods that combines nonlocal models and local models. The first idea comes from peridynamics model with a heterogeneous nonlocal horizon(interaction length). By allowing a smooth change of horizon from nonzero to zero, we effectively have a churn seamless coupling of nonlocal and local models. Another idea is borrowed from the quasicontiumm method in the atomistic-to-continuum coupling which leads to a way to get a well-posed model that passes the patch test.

Hydraulic fracture regimes and their applications

Egor V. Dontsov, W.D. Von Gonten Laboratories, Houston, USA. Email: egor.dontsov@wdvglab.com and University of Houston, Houston, USA. Email: edontsov@central.uh.edu.

Hydraulic fracturing is a technique for stimulating oil and gas wells, in which a viscous fluid is injected deep into a rock formation to produce high conductivity channels that facilitate flow of hydrocarbons back to the surface. Even for simple fracture geometries, such as plane strain or axisymmetric fractures, the solution features an interesting behavior due to interplay of physical mechanisms associated with the fluid viscosity, fracture toughness, and fluid leak-off into the formation. In particular, it is known that there are four types of self-similar solutions that correspond to the so-called regimes of propagation. The latter solutions occur for some limiting parameters that correspond to domination of one physical process, such as viscosity or toughness. The global solution, on the other hand, gradually transitions from one regime (or self-similar solution) to another in time. The "structure" of the global solution in the parametric space is investigated for plane strain and radially symmetric fractures. That is, location of the solution relative to the limiting cases is obtained for any problem parameters. Propagation of multiple closely spaced hydraulic fractures with limited entry design is studied with respect to the regime of propagation. It is found that the fracture shapes evolve from "pancakes" to a "flower" during the transition from the viscosity to the toughness dominated regimes. In the former case, all fractures are mostly radially symmetric and have approximately the same size. At the same time, the fracture "flower" is formed when each fracture has a shape of a petal. There is almost no overlap between the fractures if one observes them from the side. This enables one to influence fracture geometry of multiple fractures in field applications by controlling the regime of propagation.

Smooth And Rough Growing Three-Dimensional Cracks

Gregory J. Rodin, Institute for Computational Engineering and Sciences The University of Texas at Austin Austin, TX 78712 gjr@ices.utexas.edu

First, we consider general 3-D smooth crack fronts, for which the standard K-fields can be unambiguously developed. We show that in this case crack growth governing equations can be formulated in the local normal plane, that is, the plane perpendicular to the local tangent vector. This allows us to adopt any 2-D crack growth criterion for 3-D cracks. Second, we consider crack growth involving fragmentation and multi-fractal patterns, observed in experiments on cracks subjected to combined Mode I and III conditions.

Subsonic penetration of a thin rigid body into an elastic medium with crack-like cavities ahead and behind the body

Yuri A. Antipov, Department of Mathematics LSU

The interaction of a thin rigid inclusion with a finite crack is studied. Two plane problems of elasticity are considered. The first one concerns the case when the upper side of the inclusion is completely debonded from the matrix, and the crack penetrates into the medium. In the second model, the upper side of the inclusion is partly separated from the matrix, that is the crack length is less than the inclusion length. It is shown that both problems are governed by a singular integral equation of the same structure. Derivation of the closed-form solution of this integral equation is the main result of the paper. The solution is found by solving the associated vector Riemann-Hilbert problem with the Chebotarev-Khrapkov matrix coefficient. A feature of the method proposed is that the vector Riemann-Hilbert problem is set on a finite segment, while the original Khrapkov method of matrix factorization is developed for a closed contour.

Mathematical and computational aspects of fracture, Part II

Organizers: Blaise Bourdin, (bourdin@lsu.edu) LSU, and Robert Lipton, (lipton@lsu.edu) LSU

A peridynamics study of predicting ductile fracture in additively manufactured metal

Masoud Behzadinasab and John T. Foster , Department of Aerospace Engineering and Engineering Mechanics, University of Texas at Austin

Prediction of dynamic ductile fracture in metallic alloys with complex geom- etry involves lots of challenges. Damage accumulation along the plastic loading path governs the fracture initiation in ductile materials; thus, it is vital to ac- curately simulate deformations of a malleable structure before predicting its failure behavior. Over the past two decades, the peridynamic theory has been exploited for modeling dynamic problems involving fracture. While peridynam- ics has broadly been applied to brittle materials, its robustness in modeling ductile fracture has largely remained untested. Sandia Fracture Challenge 2017 (SFC3) provided an opportunity for the mechanics community to unbiasedly as- sess their modeling capability in predicting deformations and failure behavior in additively manufactured metal. We participated in the challenge to investigate the accuracy of the state-of-the-art of peridynamic modeling of ductile fracture. Recently Foster et al. (2017) proposed a finite deformation correspondence framework for peridynamics by incorporating classical elasto-plasticity theories. Tupek et al. (2013) also introduced a constitutive damage modeling approach for peridyanmics to take advantage of the well-established classical failure theories. A material model corresponding to the finite strain elasto-plasticity theory of Simo (1988) and a damage model associated with Johnson-Cook failure criterion (1985) have been implemented in Peridigm, an open-source massively-parallel computational peridynamics code. This framework has been applied to SFC3. The model was first calibrated by the data provided by Sandia National Lab- oratories. Following that, a blind prediction was performed on the challenge problem. Simulation results are compared with the experiments to assess the approach.

The Dirichlet Problem for a Nonlocal System of Equations Related to Peridynamics Jimmy Scott, Department of Mathematics University of Tennessee

The focus of this talk is the Dirichlet problem associated with a strongly coupled system of nonlocal equations. The system of equations comes from a linearization of a model of peridynamics, a nonlocal model of elasticity. The system is a nonlocal analogue of the Navier-Lame system of classical elasticity. The leading operator is an integral operator characterized by a distinctive matrix kernel which is used to couple differences of components of a vector field. In this talk we will present well posedness of the system of equations and demonstrate optimal interior Sobolev regularity of solutions. We apply Hilbert space techniques for well posedness. This talk presents joint work with Tadele Mengesha.

Crack nucleation in variational phase-field models of fracture Blaise Bourdin , Department of Mathematics LSU

Phase-field models of fracture have typically been constructed as regularizations of Francfort and Marigo's variational models of brittle fracture. As such, a significant part of its mathematical analysis focuses on establishing their limit as the regularization parameter vanishes. In the last few years however, there has been a growing trend of constructing then characterizing variational phase-field models of fracture as gradient-damage models. In this presentation, I will first recall the construction of variational phase field models as gradient damage models, and recall from of their properties. I will show how the identification of the regularization parameter as a material characteristic length is consistent with the literature. I will then show that this approach allows quantitative prediction of crack nucleation at weak and strong singularities and stress concentration, as well as size effects, both of which cannot be properly accounted within the LEFM framework. I will illustrate my claim with validation and verification numerical simulations.

Convergence results for finite element and finite difference approximation of nonlocal fracture models

Prashant K. Jha, Department of Mathematics, LSU

We consider a nonlocal fracture model and study the convergence properties of the finite element and the finite difference approximation. The model is a nonlinear state-based peridynamic model. We show the well-posedness of the model and show a priori convergence rate for the finite element and the finite difference approximation. We perform numerical experiments with fracture and numerically compute the rate of convergence. We show the good agreement between theoretical convergence rate and numerically computed convergence rate. The peridynamic model is designed such that the nonlocal fracture energy and the classical Griffith?s fracture energy agree for any given size of horizon. This is demonstrated through numerical examples.

Numerical approximation of fractional differential equations

Organizers: Andrea Bonito, (bonito@math.tamu.edu) Texas A&M, and Robert Lipton, (lipton@lsu.edu) LSU

Consistent traction boundary conditions for nonlocal models

Xiaochuan Tian, University of Texas, Austin

It is a recurring problem in scientific computing to design efficient numerical algorithms for a problem imposed on unbounded domains. Such problem is multiscale in its nature since the fundamental difficulty usually roots in the existence of two widely separated spatial scales. The first is a large spatial scale representing the size of the space or the material body. The second is much smaller spatial scale of interest, for example, the part of the material body where crack or fracture actually happens or the the part of the body on which non-zero external forces exert. In this talk, we introduce a consistent nonlocal traction boundary condition for static nonlocal problems so that the computation limited to a finite domain still approximates the correct result for an unbounded domain. In addition, such nonlocal traction boundary problem preserves patch constancy and is shown to be second order convergent to the corresponding local Neumann boundary problem.

Rational approximations to functions involving fractional powers of elliptic operators. Joe Pasciak, Department of Mathemaics, Texas A&M University, College Station

I will consider rational approximations to functions involving fractional powers of elliptic operators. We consider three problems, the first approximating $L^{-\alpha}f$ where L is an unbounded elliptic operator corresponding to a symmetric second order PDE with homogeneous boundary conditions and $\alpha \in (0, 1)$. The second involves a parabolic problem with a spatial operator given by $L^{\alpha}u$. The third problem replaces the time derivative in the second by a fractional Caputo derivative in time. For the second and third problem, we seek to approximate the solution at the end time T without time stepping. We discretize these problems using finite elements. The solutions of the resulting discrete problems can be expressed by the Balakrishnan formula in the case of the first problem and Dunford Taylor integrals for the second and third. These integrals are subsequently approximated using sinc quadratures. Results from Bonito-Pasciak (2015), Bonito-Lei-Pasciak (2017I) and Bonito-Lei-Pasciak (2017II) show that these sinc quadratures converge exponentially. In this talk, I consider using instead best uniform rational approximations (BURA) to related functions on [0,1]. The results for the sinc quadratures immediately imply exponential convergence bounds for the BURA approximation. Although, the development BURA for the first problem is classical, BURA approximations for problems (2) and (3) are less common. We shall present computational results illustrating their feasibility and discuss some of the problems encountered obtaining them.

Fractional phase field crystal modelling

Zhiping Mao, Division of Applied Mathematics Brown University

We propose a fractional phase field crystal (FPFC) model by replacing the standard gradient term in the original PFC free energy with a fractional order gradient, with the fractional exponent β chosen to fit the experimental structure factor. The FPFC model is shown to preserve the basic features of the original PFC model corresponding to different physical states and gives the same phase diagram as the original model, irrespective of the choice of fractional order β . However, it is shown that the FPFC model predicts a non-linear elastic behaviour and, more importantly, that there are no constraints on the bulk constants (as was the case with the original PFC model). It is shown that the FPFC equation admits a unique solution and a numerical scheme for its approximation is developed, which is used to compute the predicted values of the grain boundary energy at an internal interface, which are then compared with experimental observations.

Weighted Sobolev regularity and rate of approximation of the fractional obstacle problem Juan Pablo Borthagaray, Department of Mathematics University of Maryland

We consider the obstacle problem for the integral fractional Laplacian on bounded domains. Our goal is to obtain convergence rates for nite element discretizations of such a problem. Independently of the smoothness of the domain and the data, solutions to fractional differential equations possess limited regularity near the boundary of the domain. In order to enhance the order of convergence of the nite element approximations, we introduce suitably dened weighted fractional Sobolev spaces, where the weight is a power of the distance to the boundary, and obtain regularity results in these spaces. These bounds then serve us as a guide in the design and analysis of an optimal nite element scheme over graded meshes in two-dimensional domains.

Computation of Fractional Minimal Graphs

Wenbo Li, Department of Mathematics University of Maryland

We consider a problem of computing fractional minimal surfaces that are graphs of functions in two dimensions. This problem corresponds to a nonlinear version of fractional diffusion equation with Dirichlet boundary condition. We use finite element method to compute the solutions and establish a convergence result under certain assumptions. We show that our numerical method inherits properties from its counterpart for classical minimal surfaces. We also present and discuss several interesting numerical examples. This is a joint work with Ricardo H. Nochetto and Juan Pablo Borthagarayat University of Maryland.

Numerical geometric PDE, Part I

Organizers: Alan Demlow, Shawn Walker

A mixed finite element method for 4th order elliptic problems on surfaces Shawn W. Walker, Louisiana State University

Elliptic problems on surfaces appear in a variety of contexts, such as surfactant problems, surface diffusion, and thin film flows. The operator that appears in many surface elliptic problems is the 2nd order Laplace-Beltrami operator (i.e. the surface version of the classic Laplace operator). Many methods and analysis have been given for discretizing the Laplace-Betrami operator, e.g. surface finite elements, level set methods, etc. However, much less is known for 4th order elliptic problems on surfaces. In this work, we extend the Hellan-Herrmann-Johnson (HHJ) mixed method for plate bending problems to solving 4th order problems on manifolds with a given metric. The non-conforming approach of HHJ (with mesh dependent norms) is used to prove well-posedness and derive an error estimate. As a by-product, we show that the HHJ method does not suffer from the Babuska paradox. This framework naturally leads to surface finite elements, where the metric is induced by the embedding. Examples are given illustrating the method on closed and open surfaces, with different types of boundary conditions.

A posteriori error estimates for the Laplace-Beltrami operator on parametric surfaces Alan Demlow, Texas A&M University

Surface finite element methods (SFEM) exhibit two sources of error, a standard Galerkin finite element error and a geometric error arising from approximation of the given surface by a discrete counterpart. A posteriori error estimates for SFEM require accurate and computable measurement of the geometric error, which in turn requires that considerable geometric information about the underlying surface be practically computable. It is thus advantageous to allow for maximum flexibility in representing the surface. However, previous a posteriori estimates using general parametric surface representations are suboptimal by one order on sufficiently smooth surfaces. Estimates optimally reflecting the geometric error instead employ the closest point projection, which is defined using the signed distance function. The closest point projection is often unavailable or inconvenient to use computationally, so a posteriori estimates based on it have notable practical limitations. We merge these two perspectives by assuming practical access only to a general parametric representation of the surface, but using the distance function as a theoretical tool. Using this approach we derive sharp estimators for the geometric error.

FEM approximation of eigenvalue problems on surfaces Justin Owen, Texas A&M University

We discuss a priori estimates for Surface FEM (SFEM) approximation of eigenvalue clusters of the Laplace-Beltrami Operator. Unlike the approximation of eigenvalue clusters on flat domains, the SFEM approach for surfaces introduces a geometric source of error which comes from approximating the surface. The effects of these geometric consistency errors will be discussed. If time permits, we will also discuss a posteriori error estimation and adaptivity for approximating the eigenfunctions of the Laplace-Beltrami operator.

A finite element method for surface Navier-Stokes flows Vladimir Yushutin, University of Houston

Multiphase problems that arise in modeling of emulsions, foams, biological membranes and some other areas feature two-phase flows of bulk fluids coupled with a two-dimensional flow of matter along the curved interface between the phases. The later can be modeled by fluid systems posed on manifolds. In this talk we present a trace finite element method for solving the Navier-Stokes equations posed on a smooth closed surface. TraceFEM builds on finite element spaces defined on a fixed, surface-independent background mesh which consists of shape-regular tetrahedra. We use stabilized P_1 - P_1 pair for velocity and pressure and P_2 level-set functions for geometric approximation. Tangentiality of the velocity vector field is enforced by a penalty term. Altogether this results in a second order method for solving the surface fluid problem. Theoretical and numerical results are presented to illustrate the properties of the method.

Numerical geometric PDE, Part II

Organizers: Alan Demlow, Shawn Walker

Hybrid methods for geometric PDEs

Ari Stern, Washington University in St. Louis

Many geometric PDEs have local properties, such as symmetries and conservation laws, that one might wish a numerical method to preserve. With classical finite element methods, it is difficult to make sense of such properties, except in a weak or averaged sense. I will discuss how hybrid finite element methods, based on non-overlapping domain decomposition, provide a natural framework for talking about such properties. Specifically, I will discuss some recent results obtained by applying this approach to the multisymplectic conservation law for Hamiltonian PDEs (joint with Robert McLachlan), as well as to gauge symmetry and charge conservation in Maxwell's equations and Yang-Mills theory (joint with Yakov Berchenko-Kogan).

Geometric transformations of finite element methods Martin Light, University of California, San Diago

Martin Licht, University of California, San Diego

We discuss finite element methods for partial differential equations on manifolds. Our approach transforms partial differential equations over tensor fields from a physical manifold to parametric coordinate charts. The parametric problems involve smooth coefficients, which lead to a variational crime in practical finite element methods. Only recent results in approximation confirm optimal error estimates. In this talk we use the case of Euclidean domains as a demonstrative example and relate our approach to computational practices in engineering and physics. Moreover, extensions to the calculus of finite element differential forms are discussed.

Exact smoothed piecewise polynomial sequences on Alfeld splits Johnny Guzman, Brown University

We develop exact polynomial sequences on Alfeld splits in any spatial dimension and any polynomial degree. An Alfeld split of a tetrahedron is obtained by connecting the vertices of an n-simplex with its barycenter. We show that, on these triangulations, the kernel of the exterior derivative has enhanced smoothness. Byproducts of this theory include characterizations of discrete divergence-free subspaces for the Stokes problem, commutative projections, and simple formulas for the dimensions of smooth polynomial spaces. This is joint work with Guosheng Fu and Michael Neilan.

Numerical methods for bilayer plate models Andrea Bonito, Texas A&M University

The bending of bilayer plates is a mechanism which allows for large deformations via small externally induced lattice mismatches of the underlying materials. Its mathematical modeling consists of a geometric nonlinear fourth order problem with a nonlinear pointwise isometry constraint and where the lattice mismatches act as a spontaneous curvature. A gradient flow is proposed to decrease the system energy and is coupled with finite element approximations of the plate deformations based on either Kirchhoff or discontinuous Galerkin finite elements. In this talk, we will focus on the advantages and incovenients of each method and discuss open problems.

Numerical geometric PDE, Part III

Organizers: Alan Demlow, Shawn Walker

Q-tensor model for nematic liquid crystals with variable degree of orientation Juan Pablo Borthagaray, University of Maryland (College Park)

We consider the uniaxial Landau-de Gennes Q-tensor model for nematic liquid crystals with variable degree of orientation. In this model, the liquid crystal molecule distribution is given by a rank-one tensor and its degree of orientation. The tensor field accounts for the head-to-tail symmetry of the molecules: the states of director n and ?n are indistinguishable. The Euler-Lagrange equations for the minimizer contain a degenerate elliptic for the rank-one tensor, which, in particular, allows for half-integer defects to have finite energy. We present a structure preserving discretization of the liquid crystal energy with piecewise linear finite elements that can handle the degenerate elliptic part without regularization, and show that it is consistent and stable. We prove Gamma-convergence of discrete global minimizers to continuous ones as the mesh size goes to zero. We develop a quasi-gradient flow scheme for computing discrete equilibrium solutions and prove it has a strictly monotone energy decreasing property.

Generalized Approximate Static Condensation Method for a Heterogeneous Multi-Material Diffusion Problem

Alexander Zhiliakov, University of Houston

We introduce an approximate static condensation algorithm for solving diffusion problem in the mixed form on polyhedral meshes non-aligned with material interfaces. The method locally relies on mimetic finite difference approach and efficiently handles discontinuous material interfaces generated by interface-reconstruction algorithms. We prove that the underline system matrix is SPD, and demonstrate convergence results for several benchmarks, both stationary and time-dependent.

Optimal control of grain boundary motions

Harbir Antil, George Mason University

We propose a semi-discrete numerical scheme and establish well-posedness of a class of parabolic systems. Such systems naturally arise while studying the optimal control of grain boundary motions. The latter is typically described using a set of parabolic variational inequalities. We use a regularization approach to deal with the variational inequality. The resulting optimization problem is a nonconvex nonlinear programming problem. In the current work we are first analyzing the adjoint system corresponding to the regularized optimal control problem. Such an adjoint system is a set of highly coupled parabolic equations and proposes significant analytical and numerical challenges. We establish well-posedness of this system. In addition, we design a provably convergent semi-discrete (time discrete spatially continuous) numerical scheme to solve the system. We have developed several new tools during the course of this work that can be applied to a wider class of coupled systems.

Pointwise error estimates for a two-scale method for the Monge-Ampère equation Wenbo Li, University of Maryland (College Park)

We derive optimal error estimates in the max norm for the two-scale method for the Monge?Ampre equation introduced by Nochetto, Ntogkas, and Zhang for dimension greater than or equal to 2. The technique hinges on the discrete comparison principle and the construction of suitable discrete barrier functions. We apply this technique to classical solutions with either Hlder or Sobolev regularity. This is a joint work with Ricardo H. Nochetto at University of Maryland.

High-order accurate numerical methods for multi-physics problems Organizers: Longfei Li

Error estimates for immersogeometric methods with application to bioprosthetic heart valves Yue Yu, Lehigh University

we investigate the analysis and computation of a recent numerical method for enforcing fluidstructure interaction (FSI) kinematic constraints in the immersogeometric framework for cardiovascular FSI. In the immersogeometric framework, the structure is modeled as a thin shell, and its influence on the fluid subproblem is imposed as a forcing term. This force has the interpretation of a Lagrange multiplier field supplemented by penalty forces, in an augmented Lagrangian formulation of the FSI kinematic constraints. Because of the non-matching fluid and structure discretizations used, no discrete inf-sup condition can be assumed. To avoid solving (potentially unstable) discrete saddle point problems, the penalty forces are treated implicitly and the multiplier field is updated explicitly. Moreover, to improve the stability and conservation of the method, in a recently-proposed extension we projected the multiplier onto a coarser space and introduced the projection-based immersogeometric method.

In this talk we formulate this projection-based algorithm for a linearized parabolic model problem in a domain with an immersed Lipschitz surface, analyze the regularity of solutions to this problem, and provide error estimates for the projection-based immersogeometric method in both the $L^{\infty}(H_1)$ and $L^{\infty}(L_2)$ norms. Numerical experiments indicate that the derived estimates are sharp and that the results of the model problem analysis can be extrapolated to the setting of nonlinear FSI, for which the numerical method was originally proposed. Lastly, we have also validated the immersogeometric method in vitro by comparing the simulation results with experimental results on a latex value in an acrylic tube, which demonstrates the applicability of the method on heart value simulations.

A stable added-mass partitioned (AMP) algorithm for elastic solids and incompressible flows Dan Serino, Rensselaer Polytechnic Institute

A robust and efficient "added-mass partitioned" (AMP) algorithm is developed for simulating fluid-structure interaction (FSI) problems involving viscous incompressible fluids and compressible elastic solids. Different applications of FSI include blood flow through arteries and aero-elastic vibrations of aircraft wings, wind turbines, and bridges. In order to efficiently resolve complex moving geometries, the fluid and solid domains are discretized using deforming composite grids (DCGs). The AMP scheme is "partitioned", so the fluid and solid are solved separately and later linked together using interface boundary conditions. Industry grade partitioned solvers are efficient, but often require sub-iterations to avoid numerical instabilities when "added-mass" effects are large (prevalent for light solids or high-resolution computational meshes). The AMP algorithm eliminates the need for sub-iterations by applying special interface conditions motivated from the wave propagation of characteristics in the solid domain. A stability analysis confirms that the AMP scheme is stable for any mass ratio of the fluid and solid. Numerical examples are showcased to verify the accuracy and robustness of the AMP algorithm.

High-Order accurate conservative finite difference methods for Vlasov equations in 2D+2V Andre Gianesini Odu, Rensselaer Polytechnic Institute

Kinetic simulation of multi-dimensional plasma waves through direct discretization of the Vlasov equation is a useful tool to study many physical interactions and is particularly attractive for situations where minimal fluctuation levels are desired, for instance, when measuring growth rates of plasma wave instabilities. However, direct discretization of phase space can be computationally expensive, and as a result there are few examples of published results using Vlasov codes in more than a single configuration space dimension. In an effort to fill this gap we have developed the Eulerian-based kinetic code LOKI that evolves the Vlasov-Poisson system in 2+2-dimensional phase space. Here we discuss the the 4th- and 6th-order accurate finite difference algorithms used in the code, and present rigorous verification of convergence rates using both manufactured solutions and extraction of physically relevant quantities of interest such as Landau damping decay rates.

A Higher-order DG/ALE partitioned approach to solving fluid-structure interaction problems Annalisa Quaini, University of Houston

We present a discontinuous Galerkin-based numerical method for solving fluid-structure interaction problems involving incompressible, viscous fluids. The fluid and structure are fully coupled via two sets of coupling conditions. The numerical approach is based on a high-order discontinuous Galerkin (with Interior Penalty) method, which is combined with the Arbitrary Lagrangian-Eulerian approach to deal with the motion of the fluid domain, which is not known a priori. Two strongly coupled partitioned schemes are considered to resolve the interaction between fluid and structure: the Dirichlet-Neumann and the Robin-Neumann schemes. The proposed numerical approach provides sharp resolution of jump discontinuities in the pressure and normal stress across fluid-structure and structure-structure interfaces. It also provides a unified framework for solving fluid-structure interaction problems involving nonlinear structures, which may develop shock wave solutions that can be resolved using a unified discontinuous Galerkin-based approach. We will showcase the performance of the proposed method though a series of benchmark problems and a fluid-structure interaction problem describing the flow of blood in a patient-specific aortic abdominal aneurysm before and after the insertion of a prosthesis known as stent graft.

Mathematical modeling in ecology and epidemiology, Part I

Organizers: Hayriye Gulbudak (University of Louisiana at Lafayette), Mac Hyman (Tulane University)

Resource-driven encounters among consumers and the implications for the spread of disease Scott A McKinley, Tulane University

Animals share a variety of common resources, which can be a major driver of conspecific encounter rates. In this talk, we will look at a spatially explicit model for resource visitation behaviour in order to examine how changes in resource availability can influence the rate of encounters among consumers. Using simulations and asymptotic analysis, we will see a somewhat unexpected non-monotonic relationship resource availability and consumer conspecific encounters, and

interpret the results in terms of basic reproduction number for a population that is susceptible to a novel pathogen. We validate certain aspects of the movement model using observations of large aggregations of black-backed jackals at carcasses generated by seasonal outbreaks of anthrax among herbivores in Etosha National Park, Namibia. (This is joint work with Rebecca K Borchering, Steven E Bellan, Juliet RC Pulliam, and Jason M Flynn.)

Network Model for Ecology of Virus and Immune Response during HIV Infection Cameron Browne, ULL

The dynamics of virus and immune response within a host can be viewed as a complex and evolving ecological system. For example, during HIV infection, an array of CTL immune response populations recognize specific epitopes (viral proteins) presented on the surface of infected cells to effectively mediate their killing. However HIV can rapidly evolve resistance to CTL attack at different epitopes, inducing a dynamic network of viral and immune response variants. We consider models for the network of virus and immune response populations. Our analysis provides insights on viral immune escape from multiple epitopes. In the "binary allele" setting, we prove that if the viral fitness costs for gaining resistance to each of n epitopes are equal and multiplicative, then the system of 2^n virus strains converges to a "perfectly nested network" with less than n+1 persistent virus strains. Overall, our results suggest that immunodominance is the most important factor determining viral escape pathway of HIV against multiple CTL populations.

Viral dynamics revisited: partial degeneracy and spatial heterogeneity Xiang-Sheng Wang, ULL

We study a general viral infection model with spatial diffusion in virus and two types of infection mechanisms: cellfree and cell-to-cell transmissions. The model is a partially degenerate reaction-diffusion system, whose solution map is not compact. We identify the basic reproduction number R_0 and explore the properties of R_0 when the virus diffusion parameter varies from zero to infinity. Moreover, we demonstrate that the basic reproduction number is a threshold parameter for the global dynamics of our model system: the infection and virus will be cleared out if $R_0 \leq 1$, while if $R_0 > 1$, the infection persists and the model admits a unique positive infection steady state which is globally attractive. Numerical simulation supports our theoretical results and suggests an interesting phenomenon: boundary layer and internal layer may occur when the diffusion parameter tends to zero.

Effects of host infectivity and susceptibility on disease emergence in stochastic multigroup models with applications to emerging and re-emerging infectious diseases Aadrita Nandi, Texas Tech

A serious concern to public health is emerging and re-emerging infectious diseases, including those of zoonotic origin such as SARS, MERS and Ebola and re-emerging diseases such as measles and pertussis. Amplification and spread of infection in several emerging diseases have been attributed to highly infectious individuals known as superspreaders. Vaccine waning or lack of vaccine protection are some of the reasons for disease outbreaks in re-emerging diseases. We apply Markov chain models and branching process approximations in the setting of multigroup models to investigate the probability, final size and duration of an outbreak when transmission depends on either host infectivity or host susceptibility. The models show that probability of a major disease outbreak is greater when initiated by a superspreader as compared to a nonsuperspreader or when initiated by a susceptible individual with a long duration of infection as compared with a short duration. Also, the examples illustrate that when host infectivity is a major contributing factor to outbreaks in emerging diseases, as opposed to host susceptibility, the overall probability of an outbreak is smaller but the final size is much greater.

Mathematical modeling in ecology and epidemiology, Part II Organizers: Hayriye Gulbudak, Mac Hyman

Two-strain multi-scale dengue model structured by dynamic host antibody level Hayriye Gulbudak, ULL

Infection by distinct Dengue virus serotypes and host immunity are intricately linked. In particular, certain levels of cross-reactive antibodies in the host may actually enhance infection severity. The coupled immunological and epidemiological dynamics of Dengue calls for a multi-scale modeling approach. In this work, we formulate a within-host model which mechanistically recapitulates characteristics of antibody dependent enhancement (ADE) in Dengue infection. The within-host scale is then linked to epidemiological spread by a vector-host partial differential equation

model structured by host antibody level. The coupling allows for dynamic population wide antibody levels to be tracked through primary and secondary infections by distinct Dengue strains, along with waning of cross-protective immunity after primary infection. Analysis of both the within-host and between-host systems are conducted. Stability results in the epidemic model are formulated via basic and invasion reproduction numbers as a function of immunological variables. Additionally, we develop numerical methods in order to simulate the multi-scale model and assess the influence of ADE on disease spread and burden in the population. Finally, the model is extended to include vaccination so that evaluation of vaccine effect on antibody level can be extrapolated to epidemiological impacts.

An Agent Based Model for the Transmission of Chlamydia through a Heterosexual Network Embedded in Social Network Mac Hyman, Tulane University

The structure of a sexual network plays an important role in how sexually transmitted infections (STIs) spread. Generating an ensemble of networks which mimics the real-world is crucial to evaluating robust mitigation strategies for public health staff to control STIs. Most of the current algorithms to generate sexual networks only use sexual activity data, such as number of partners per month, to generate the sexual network. Real-world networks also depend on biased mixing based on age, location, and social and work activities. We describe an approach to use a broad range of social activity data to generate possible heterosexual networks of partners. This social network accounts for the correlations between people of different ages, lining in different locations, economic status, and other demographic factors. We evaluate the effectiveness of condom use, notifying the partners of infected people that they may be infected, as well as the effectiveness of notifying nonsexual social friends that the infection is prevalent. We observed that when a person is found to be infected, then a combination of both increased condom use, sexual partner and social friend notification is an effective approach to mitigation an epidemic. This research is in collaboration with Asma Azizi, Patricia Kissinger, Bryan Lewis, and Zhuolin Qu.

Effects of Distribution for the Time-Since-Infection and Risk Change on a Vector-Borne Model Li Guan, Tulane University

Most mathematical epidemic models assume that rate people recover from an infection is independent of the length of time they have been infected. This can be a poor approximation for most diseases. We explore the qualitative and quantitative impact of including a more realistic distribution of the time from infection to recovery in mathematical models on the predicted course of an epidemic. A theoretical analysis of the equations and numerical simulations are used to demonstrate how the distributed incubation times have a strong impact on the dynamics of the epidemics. We then develop a hybrid ODE-PDE non-local model for the transmission and spread of vector-borne infectious diseases to include the time since infection factor for vector and we also consider the risk change for host. We derive the theoretical solution for the model and prove its Well-posedness. We develop the formula of the basic reproduction number with respect to the model parameters for this vector borne model to compare with the classic ODE model with constant recovery rate and no behavior change. We include the numerical simulation for the dynamics of the susceptible, infectious populations for vectors as a function of time and time since infection.

Antibody-mediated immobilization of virions in mucus

Melanie Jensen, Tulane University

Antibodies have been shown to hinder the movement of Herpes Simplex Virus (HSV) virions in cervicovaginal mucus (CVM). It has been hypothe- sized that virion-surface-bound antibodies individually have a weak affinity for mucin fibers and, together, create a virtual affinity between a virion and its surrounding mucosal environment. It has not been clear, though, whether the hinderance of the virion movement is incremental with increasing antibody concentrations, or if the effect is an all-or-nothing dichotomy between free dif- fusion and near-complete immobilization. Based on single particle tracking experiments, in this work we argue for the latter case. While populations of freely diffusing and immobilized particles are seen across all samples, we introduce a Bayesian switch-point detection method to show that there are very few, if any, switches between these states on the experimental time scale of twenty seconds. Moreover, using a continuous-time Markov Chain model for the virion-antibody-mucin kinetics, we argue that slow switching implies that virion immobilization requires cooperation by many simultaneously bound antibodies.

Mathematics in oil and gas exploration and production, Part I

Organizers: Ipsita Gupta, Monika Valjak; igupta@lsu.edu, monika.valjak@chevron.com

The purpose of this session is to highlight the application of Mathematics in every day oil and gas activities. In the first part of the session, we present a half-'n'-half mix of academia and industry talks from reservoir, drilling and production applications, including conventional and unconventional resources. The second part of the session focuses on the application of data science and data analytics to both upstream and downstream sectors.

Geostatistic modeling applications in Deepwater Gulf of Mexico reservoirs

Monika Valjak and Francisco Correa Mora, Chevron; Monika.Valjak@chevron.com,

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We present an overview of reservoir modeling in the oil industry, with emphasis on the techniques and algorithms employed to build static earth models during the different phases of an oil field. Thanks to recent significant computational efficiency improvements, earth models have become a tool to represent reservoirs during the appraisal and development phases of an oil field. Earth models can be used as a secondary method to estimate oil in place during the appraisal phase, and as a primary method to estimate hydrocarbon ultimate recovery when coupled with dynamic simulations that match production history. During the appraisal phase, well data is sparse on deep water fields; therefore, earth scientists use analog knowledge andgeostatistical techniques to create a geologically reasonable suite of earth models to appropriately assess the uncertainty of hydrocarbons in place. When the development phase of the field starts, more data becomes available. Production rates and pressures are used to calibrate the initial suite of models during history matching, reducing the uncertainty range. The uncertainty evaluations are typically done using design of experiments methods and Monte Carlo simulations. Some examples of current challenges include to develop algorithms that propagate properties using both physical laws and geostatistics, or algorithms for facies classification of well logs that include uncertainty of the measurements, and better implementation of screening tools.

Ensemble-based Methods for History Matching and Production Optimization

Yanfen Zhang and Hemant Phale, Chevron; yanfen.zhang@chevron.com, hemantphale@chevron.com

History matching is a critical step of calibrating dynamic reservoir models to available historical production/pressure data. Reservoir models that are history matched while preserving geological realism form a basis for generating reliable production forecast and to carry out any type of production optimization for maximizing value of the asset. A number of methods including Design of Experiments (DoE), Genetic Algorithm (GA), and gradient-based adjoint methods have been proposed for history matching application. Most often, DoE is selected as a history matching method due to the simplicity of its application. The other history matching techniques such as adjoint methods require better understanding of underlying mathematical principles and often require large computational resources. As a result, the application of these techniques has been infrequent. In the last couple of decades, the ensemble-based methods have established themselves as viable history-matching and production optimization techniques suitable for reservoir models with large number of uncertainty variables. With the computational resources becoming increasingly cheaper, techniques such as ensemble-based methods have huge potential to become a routine history matching/optimization method in oil and gas industry. This talk will primarily focus on the underlying mathematical formulation of ensemble-based methods and will cover a few examples to demonstrate the applicability of these techniques for history matching and production optimization, and production optimization.

Gas flow in non-Newtonian drilling fluids - A CFD approach Hope Asala and Ipsita Gupta, LSU; hikech1@lsu.edu, igupta@lsu.edu

During exploratory well drilling, uncontrollable gas influx and migration into the wellbore may lead to undesirable well blowouts. Gas migration in non-Newtonian drilling fluids is a multiphase, multi-physics problem that is still poorly understood. In this work we employ a computational and analytical approach to understand and model transient flow regime development when a gas kick occurs. Using ANSYS Fluent, a 3D model is developed to simulate the dynamic behavior of pressure, volume fraction, density and viscosity as gas flows from invasion zone. Preliminary analytical (Newtonian and non-Newtonian) and numerical models are presented for analysis. Final outcome of this work will be utilized for engineering drilling fluids real-time for the mitigation of well blowout events.

Orientation of hydraulic fracture initiation from horizontal wellbores: An analytical and numerical Study

Andreas Michael(a), Jon Olson(b), and Matthew Balhoff(b), a: LSU, b: UT Austin, andreasmichael89@yahoo.com

A three-dimensional numerical analysis using a numerical simulator is performed to assess an existing orientation criterion for fracture initiation in perforated wellbores. We find that it is likely for typical treatments in horizontal wells to achieve longitudinal fracture initiation in horizontal wells as transverse initiation will only occur over a narrow window of conditions including low wellbore pressure at breakdown and low formation tensile strength.

Mathematics in oil and gas exploration and production, Part II

Organizers: Ipsita Gupta, Monika Valjak; igupta@lsu.edu, monika.valjak@chevron.com

The purpose of this session is to highlight the application of Mathematics in every day oil and gas activities. In the first part of the session, we present a half-'n'-half mix of academia and industry talks from reservoir, drilling and production applications, including conventional and unconventional resources. The second part of the session focuses on the application of data science and data analytics to both upstream and downstream sectors.

Statistical Estimation and Machine Learning of Proppant and Gravel Packs Petrophysical Properties from Pore Scale Simulations

Sultan Anbar and Mayank Tyagi, LSU, mtyagi@lsu.edu

Proppant and gravel packs are used to keep the hydraulic fractures and completions around the wellbore open. In this study, Lattice Boltzmann Method (LBM) is used for reliable estimation of flow properties of gravel packs to develop correlations for permeability and non-Darcy coefficient after establishing the simulation domain and error controls. With the Design of Experiment (DOE), 100 computer generated sphere packs are constructed by using the range of porosity and the grain size distribution parameters found in the literature to create a database for gravel packs. Permeability and non-Darcy coefficient correlations are developed with linear regression, and a novel approach, path analysis, is applied to petrophysical properties to highlight the relations between them. Artificial Neural Network (ANN) methodology is used to estimate flow properties from design parameters; grain size distribution and porosity. Estimated properties from path model and ANN show good comparison except for tortuosity due to its sensitivity to flow paths in complex pore spaces.

Cognitive machine monitoring: industrial monitoring systems using machine learning and deep learning approaches

Wenbo Zhu, Zachary Webb and Jose Romagnoli, LSU; wzhu16@lsu.edu, jose@lsu.edu

TBD

Data Driven Modeling and Predictive Analytics for Waterflooding Operations using Reservoir Simulations

Xuan Liao and Mayank Tyagi, LSU, mtyagi@lsu.edu

In this talk, Data driven modeling techniques (DDM) have been adopted as an alternative tool to predict production performance under waterflooding which is one of the most important techniques for improving oil recovery. A synthetic waterflooding dataset including production profile, operational parameters, reservoir properties and well locations is constructed using the numerical reservoir simulator. K-means clustering analysis is performed to identify internal groupings among oil producer wells. Artificial neural network (ANN) and support vector machine (SVM), and support vector regression (SVR) are used to understand the nonlinear relationships between input attributes and waterflooding production. Trained models are subsequently used to predict cumulative oil production and watercut on the unseen data samples. Clustering analysis reveal that distance to the free water level (FWL) has a dominant effect since clustering assignment is controlled by the interplay among input attributes characterizing reservoir properties and relative well locations. Good agreements between predicted outputs from models and simulation targets present the satisfactory generalization performance and predictive capabilities of ANN and SVR methods.

Artificial neural networks for unconventional reservoirs production and supply chain optimization

Jorge Chebeir, Vidyadhar Manee, Hope Asala, Jose Romagnoli and Ipsita Gupta, LSU, jchebe1@lsu.edu

This work presents a techno-economic framework for the production and distribution of unconventional resources located in the Marcellus Shale play. Reservoir simulation and data analytic techniques are coupled with a strategic planning model to determine long-term development schemes for shale gas exploitation and delivery. A 3-D reservoir model is combined with a supervised machine learning (SML) algorithm to define optimal field development

strategies for the shale gas formation. A mixed-integer non-linear programming (MINLP) model incorporates these field development strategies to define the optimal design and operation of the surface facilities during the planning horizon of the project. Additionally, long-short term memory (LSTM) neural networks are utilized for the prediction of certain exogenous parameters such as the natural gas demand and the water availability in the region.

Imaging and inverse problems, Part I Organizers: Tan Bui-Thanh, UT Austin, ICES

Inverse problems (and imaging) are ubiquitous in engineering and sciences. Inverse problems are the task of inferring unknowns (e.g. parameters) that can only be indirectly observed via a solution of a mathematical problem (forward problem): i.e. inverse problems are mathematical techniques that combines observational data and mathematical models to calibrate unknown parameters. This mini-symposium collects research on mathematical methods and algorithms developed in TX-LA region to tackle challenges in large-scale and complex inverse problems.

Bayesian inversion of fault properties in two-phase flow in fractured media Ilona Ambartsumyan, Tan Bui-Thanh, Omar Ghattas, Eldar Khattatov, and Umberto Villa,

We address the inverse problem of inferring fault properties from well observations of two-phase flow in fractured media. The two-phase flow model in its mixed form is modified to account for fractures in the medium, thus permitting pressure discontinuities across the faults. We adopt a Fully Implicit Method (FIM) for solving the variational problem that arises, providing a path to computing adjoint-consistent first and second order derivatives of observables with respect to fault parameters.

We formulate the inverse problem in the framework of Bayesian inference, and seek the solution as a posterior probability distribution characterizing uncertainty in the inferred fault parameters. The full Bayesian solution using black box model is often prohibitive for expensive forward models such as two-phase flow. We begin by constructing a Laplace approximation of the posterior, i.e. a Gaussian approximation of the posterior with mean given by the maximum a posteriori (MAP) point, and covariance given by the inverse of the Hessian of the negative log posterior. The MAP point is found by a scalable deterministic adjoint-based inexact Newton-CG optimization algorithm, while the inverse Hessian is made tractable by a low-rank approximation of the log likelihood, constructed via a randomized eigensolver. Finally, the Laplace approximation is employed as a preconditioner for a preconditioned Crank-Nicolson MCMC method to sample the posterior distribution. The resulting exploitation of the smoothness, geometry, and sparsity of the posterior leads to scalable algorithms whose cost, measured in the forward PDE solves, scales independent of the parameter and data dimensions, and depends only on the number of informed modes of the parameters. The inversion algorithms employed are provided by the hIPPYlib library.

Seismic modeling and imaging in elastic and viscoacoustic media Hejun Zhu,

Reverse time migration (RTM) is one of the most accurate imaging tools available in the energy industry, which allows us to take complicated wave physics into account, and map complex Earth?s structures in great details. In the first part of this talk, I will present elastic RTM technologies developed in the UTD Seismic Imaging Laboratory for delineating reflectivity distributions in complex subsurface environment, as well as their applications to imaging earthquake rupture processes. Synthetic results demonstrate that the new technologies allow us to deliver better PS migration images, and characterize complicated temporal and spatial evolution of earthquake rupture processes. Attenuation is an energy dissipation mechanism that manifests itself in the form of physical dispersion and amplitude dissipation. The combination of these two effects can significantly distort the traveltimes and amplitudes of seismic wavefields. Without properly taking these effects into account, we will get inaccurate results in seismic modeling, migration and inversion, which will lead to wrong geological interpretation and reservoir characterization. In the second part of this talk, I will present technologies to promote seismic modeling and imaging in viscoacoustic media. The developed viscoacoustic wave equation allows us to: (1) separate physical dispersion and amplitude dissipation, which can be utilized in Q-compensated RTM; (2) explicitly incorporate Q in the wave equation, which can be utilized in Q full waveform inversion.

Efficiently evaluating rare event probabilities Siddhant Wahal,

We consider the problem of estimating rare event probabilities, focusing on systems whose evolution is governed by differential equations with uncertain input parameters. If the system dynamics is expensive to compute, standard sampling algorithms such as the Monte Carlo method may require infeasible running times to accurately evaluate these probabilities. To address this problem, we recently proposed an importance sampling algorithm called the Bayesian Inverse Monte Carlo (BIMC) method. BIMC arrives at an importance sampling density by solving a fictitious Bayesian inverse problem. The solution of the inverse problem yields a posterior density, a local Gaussian approximation to which serves as the IS density. Although, BIMC produces an optimal IS density under certain conditions, it is also prone to fail in systems with highly non-linear dynamics. This talk will review the BIMC method, and discuss our attempt at formulating an adaptive algorithm that is applicable to highly non-linear problems. We will conclude by drawing connections with other open problems in Bayesian inverse problems and identifying directions for future research.

Imaging and multiple removal via model order reduction Alexander V. Mamonov joint with L. Borcea, V. Druskin and M. Zaslavsky,

We introduce a novel framework for imaging and removal of multiples from waveform data based on model order reduction. The reduced order model (ROM) is the orthogonal projection of the wave equation propagator (Green's function) on the subspace of discretely sampled time domain wavefield snapshots. Even though neither the propagator nor the wavefields are known in the bulk, the projection can be computed just from the knowledge of the waveform data using the block Cholesky factorization. Once the ROM is computed, its use is twofold.

First, the projected propagator can be backprojected to obtain an image. ROM computation implicitly orthogonalizes the wavefield snapshots. This highly nonlinear procedure differentiates our approach from the conventional linear migration methods (Kirchhoff, RTM). It allows to resolve the reflectors independently of the knowledge of the kinematics and to untangle the nonlinear interactions between the reflectors. As a consequence, the resulting images are almost completely free from the multiple reflection artifacts.

Second, the ROM computed from the original waveform data can be used to generate the Born data set, i.e. the data that the measurements would produce if the propagation of waves in the unknown medium obeyed Born approximation instead of the full wave equation. Obviously, such data only contains primary reflections and the multiples are removed. Consecutively, existing linear imaging and inversion techniques can be applied to Born data to obtain reconstructions in a direct, non-iterative manner.

Imaging and inverse problems, Part II Organizers: Tan Bui-Thanh, UT Austin, ICES

Inverse problems (and imaging) are ubiquitous in engineering and sciences. Inverse problems are the task of inferring unknowns (e.g. parameters) that can only be indirectly observed via a solution of a mathematical problem (forward problem): i.e. inverse problems are mathematical techniques that combines observational data and mathematical models to calibrate unknown parameters. This mini-symposium collects research on mathematical methods and algorithms developed in TX-LA region to tackle challenges in large-scale and complex inverse problems.

A Scalable Approach to the Consistent Bayesian Method Brad Marvin and Tan Bui-Thanh,

We present a scalable approach to the solution of linear and nonlinear stochastic inverse problems using the recently developed consistent Bayesian method. In particular, the use of kernel density estimation in the consistent Bayesian method results in poor scaling with the dimension of the data. Our approach avoids the need for kernel density estimation and is therefore scalable with the data dimension. In addition, we adapt a randomized dimension reduction technique developed for the classical Bayesian maximum a posteriori (MAP) point method to develop a consistent Bayesian MAP point method which scales with the data, state, and parameter dimensions. We demonstrate our scalable approach on a linear inverse problem govern by an advection-diffusion equation and a nonlinear inverse problem govern by a hyperelasticity equation.

A hybrid inverse problem in the fluorescence ultrasound modulated optical tomography in the diffusive regime

Wei Li, Department of Mathematics, Louisiana State University

We investigate a hybrid inverse problem in fluorescence ultrasound modulated optical tomography (fUMOT) in the diffusive regime. We prove that the absorption coefficient of the fluorophores at the excitation frequency and the quantum efficiency coefficient can be uniquely and stably reconstructed from boundary measurement of the photon currents, provided that some background medium parameters are known. Reconstruction algorithms are proposed and numerically implemented as well. This is joint work with Yang Yang and Yimin Zhong.

A data-scalable randomized misfit approach for solving large-scale PDE-constrained inverse problems

Tan Bui-Thanh, UT Austin, ICES,

A randomized misfit approach is presented for the efficient solution of large-scale PDE-constrained inverse problems with high-dimensional data. A stochastic approximation to the misfit is analyzed using random projection theory. By expanding beyond mean estimator convergence, a practical characterization of randomized misfit convergence can be achieved. The class of feasible distributions is broad yet simple to characterize compared to previous stochastic misfit methods. This class includes very sparse random projections which provide additional computational benefit. A 5-minute Matlab demonstration will be given. The main contribution is a theoretical result showing the method guarantees a valid solution for small reduced misfit dimensions. The computational cost savings for large-scale PDE-constrained problems with high-dimensional data will be discussed. Results with different random projections will be presented to demonstrate the viability and accuracy of the proposed approach on single-phase subsurface flow problems.

Numerical PDE/ODE and HPC applications, Part I

Organizers: Don Liu

Partial differential equations (PDE) and ordinary differential equations (ODE) are most commonly used in describing phenomena in nature, besides stochastic and statistical approaches. Due to restrictions on multi-dimension, interweaving physics, and irregular domains, in addition to various complex initial and boundary conditions, exact or analytical solutions usually are unavailable. Therefore, numerical solutions to PDE and ODE become an important alternative besides experimental measurements. This session dedicates towards various methods and models for computational investigations on problems involving multi-phase, multi-physics, moving boundaries, complex geometries, transient, and sharp spatial gradients. High performance computing technologies using various parallel algorithms on architectures, e.g., CPU and GPU, are usually resorted to for the improved computational performance.

Numerical studies of complex two phase flow and convective heat transfer Don Liu, Louisiana Tech University

Three-dimensional (3D) convective heat and mass transfer is ubiquitous in nature, especially in irregular domains. Usually conservation laws about mass, momentum, and energy are used to describe phenomena and five variables, three velocities, pressure, and enthalpy (or temperature), are solved for numerical solutions. When two-phases, moving particles and fluid, are concerned, their interactions are the key to predict details of the transient coupled motion. One the other hand, for 3D printing which concerned advanced materials in manufacturing, volume or mass concentrations are also solved. Some 3D printing requires controlled laser heating and coordinated power melting and solidification. Therefore, phase change heat transfer is inevitable. In order to acquire high-order solutions in which Euclidean-normed errors decay exponentially as in spectral convergence, high-order methods in both time and space are needed to achieve the goal. The major challenges are capturing internal moving boundaries at particle-fluid interfaces and tracking solid-liquid interfaces which evolve with time and space. Algorithms and results are presented to address these problems.

Some recent development of the immersed interface method for flow simulation Sheng Xu, Southern Methodist University

The immersed interface method is a methodology for solving PDEs subject to interfaces. In this talk, I will give an overview of somerecent development of the method toward the enhancement of its robustness for flow simulation. In particular, I will present with numerical results how to capture boundary conditions on immersed rigid objects, how to adopt interface triangulation, how to model particle collisions, and how to parallelize the method for flow with moving objects. With these developments, the immersed interface method can achieve accurate and efficient simulation of a flow involving multiple moving complex objects.

Dynamic-solver-consistent minimum action method for Navier-Stokes equations Xiaoliang Wan, Louisiana State University

We developed a dynamic-solver-consistent minimum action method for Navier-Stokes equations perturbed by small noise. The minimum action method is an algorithm to capture the most probable transition path which provides information of nonlinear instability beyond the linear stability theory. Based on the same divergence-free approximation space, the dynamic solver and the minimum action method are coupled together to minimize the possible inconsistencies induced by numerical distretization such that the semi-discrete large deviation principle can be regarded as the exact large deviation principle for the semi-discrete unperturbed Navier-Stokes equations. Numerical experiments are presented.

Numerical PDE/ODE and HPC applications, Part II $\,$

Organizers: Don Liu

Modified Spline Basis Functions for Simulating PDE Systems using the Finite Element Method Lisa Kuhn, Southeastern Louisiana University

Historically, researchers have avoided using higher-degree polynomials when formulating a basis via the Finite Element due to the oscillatory behavior associated with higher degree polynomials. With the advancement of smart materials, the need for more strict smoothness requirements for basis functions is of increasing importance, while simultaneously ensuring that these functions meet the complicated prescribed boundary conditions associated with multiple component structures. The focus of this talk is an analysis of modified cubic and quintic B-spline basis functions for simulating partial differential equation systems.

Immersed boundary method in coastal hydraulic modeling Ning (Michael) Zhang, McNeese State University

Exploiting parallelisms in grid world navigation task to reduce training latency via high performance computing techniques

Steele Russell, Southeastern Louisiana University

The grid world navigation task has been employed as a proof of concept benchmark for an approachs ability to model nonlinear systems. Approximate Dynamic Programming has shown that it has the ability to generate an optimal solution to the grid world navigation task. Approximate Dynamic Programming produces an approximate solution for Bellmans equation determined by Dynamic Programming This solution, while not exact, is optimal and is achieved with reduced time and space complexity. Approximate Dynamic Programming is achieved via a Simultaneous Recurrent Neural Network (SRNN). A number of algorithms can be employed to train the SRNN. The Extended Kalman Filter has shown particular promise in training the SRNN. The advent of the General Purpose Graphics Processing Unit (GPGPU) has revolutionized hardware acceleration of parallelizable systems. The nature of the grid world navigation task lends itself to parallelization as do the calculations involved in the Kalman filter. The proposed talk will examine the impact of utilizing GPGPU hardware acceleration to exploit parallelisms in the Kalman filter algorithm and the grid world navigation task.

Image processing algorithms: progress and challenges, Part I Organizers: Hyoungsu Baek

Images in seismic exploration are providing critical information for prospecting and business decision. Cleaner and brighter seismic images are generated with the advance of acquisition and imaging technologies. However, the large volume of images degraded by incoherent and coherent noises are and will be posing huge challenges in processing and analysis/interpretation. Novel ideas from different disciplines have been very successful. In this symposium, we would like to focus on image processing algorithms: their interdisciplinary nature, algorithmic development, and data volume issues. Examples are drawn from applications to fault detection, event picking, registration, dip estimation, and wave propagation direction detection.

High Resolution 3D Dip Estimation

Hyoungsu Baek(1) and Mike Jervis(2), 1: Aramco Services Company Houston Research Center; 2: Saudi Aramco EXPEC Advanced Research Center

Dip fields computed from seismic images provide critical information for model building, migration, and interpretation. Dip estimation is challenging due to noise and interference of events as well as high dip events. We propose a novel method which works for both horizontal and vertical events. Accurate dip angles are computed efficiently by smoothing angles while initial dips are obtained from gradient vectors of input images. In order to get a smooth dip field with high resolution from the noisy initial dips, the dip angles are smoothed along the dip directions. Our method is immune to the discontinuity in the angle domain. Using the estimated dip fields, edge preserving smoothing is applied to synthetic images with controlled noises, demonstrating the robustness and accuracy of the method.

Automatic fault interpretation with optimal surface voting

Xinming Wu, Bureau of Economic Geology, University of Texas at Austin

As faults are typically recognized as reflection discontinuities in a seismic image, numerous types of fault attributes have been proposed to detect faults by measuring reflection continuities or discontinuities. However, these attributes can be sensitive to other seismic discontinuities such as noise and stratigraphic features. In addition, fault features within a fault attribute image often cannot be continuously tracked. We propose an optimal surface voting method to enhance a fault attribute image so that the noisy features (unrelated to faults) are suppressed while the fault features become cleaner and more continuous. In this method, we first automatically pick seed points from the input attribute image and use these seeds as control points to compute optimal surface patches that pass through the control points and follow globally maximum fault attribute values. We then consider all the computed surfaces as voters and define voting scores for each voter by using fault attribute values that are smoothed along the surface voter. We further collect voting scores of all the voters to compute a voting score map as a new fault attribute image, where fault features (with high scores) are much cleaner, sharper, and more continuous than those in the input attribute image. The computational cost of the method depends on the number of seed points, not the size of the seismic volume, which makes the method highly efficient. With an eight-core computer, our parallel implementation can process more than 1000 seeds in 1 second to compute the corresponding optimal voting surfaces and a final voting score map.

A robust optical flow algorithm for computing wave propagation direction Houzhu Zhang, Aramco Services Company Houston Research Center

Accurate estimation of direction of wave propagation is an important step for subsurface imaging and inversion in oil and gas exploration. Various methods have been proposed for direction computation. Among them, algorithm based on Horn-Schunck optical flow is one of the most widely used. While HS optical flow algorithm is efficient and implementation is straight forward, one outstanding issue is the lack of formulation for the regularization factor. A common practice is to choose a constant value for all the points in the computation space. This simple scheme does not work well for wave propagation in heterogeneous media. In this presentation, by investigating the analogy between Horn-Schunck equations and the Navier- Stokes equations in fluid dynamics, we propose a modified version of optical flow algorithm for computing the direction of wave propagation. The fundamental change to the conventional algorithm is the introduction of an explicit formulation of the regularization factor, which is now dependent on the local property of the wavefield. The algorithm is tested on wave propagation using complicated models from Gulf of Mexico and the results show that the proposed algorithm is more robust and accurate.

Seismic data matching

Sergey Fomel, Bureau of Economic Geology, University of Texas at Austin

Image processing algorithms: progress and challenges, Part II Organizers: Hyoungsu Baek

Computational challenges for generating balanced images

Hyoungsu Baek(1), Dongliang Zhang(2), Mohammed Mubarak(2), 1: Aramco Services Company Houston Research Center; 2: Saudi Aramco EXPEC Advanced Research Center

Surface consistent processing involves inversion of rank-deficient matrices with billions of data points. Problem sizes are getting larger as acquisition technology is advancing as well as computing power. The number of data points is expected to reach trillions soon. Algorithmic challenges are growing because of the growing size of the problem. Mathematically, the nave regularization using L1 or L2 norms is not effective. We have developed a method to incorporate extra information to constrain the underdetermined problems. Also surface consistent processing framework has been developed to handle large datasets. In this presentation, we focus on the computational and algorithmic challenges and solutions.

Least-squares horizons with local slopes and multi-grid correlations

Xinming Wu, Bureau of Economic Geology, University of Texas at Austin

Most seismic horizon extraction methods are based on seismic local reflection slopes that locally follow seismic structural features. However, these methods often fail to correctly track horizons across discontinuities such as faults and noise because the local slopes can only correctly follow laterally continuous reflections. In addition, seismic amplitude or phase information is not used in these methods to compute horizons that follow a consistent phase (e.g., peaks or troughs). To solve these problems, we have developed a novel method to compute horizons that globally fit the local slopes and multigrid correlations of seismic traces. In this method, we first estimate local reflection slopes

by using structure tensors and compute laterally multigrid slopes by using dynamic time warping (DTW) to correlate seismic traces within multiple laterally coarse grids. These coarse-grid slopes can correctly correlate reflections that may be significantly dislocated by faults or other discontinuous structures. Then, we compute a horizon by fitting, in the least-squares sense, the slopes of the horizon with the local reflection slopes and multigrid slopes or correlations computed by DTW. In this least-squares system, the local slopes on the fine grid and the multiple coarse-grid slopes will fit a consistent horizon in areas without lateral discontinuities. Across laterally discontinuous areas where the local slopes fail to correctly correlate reflections and mislead the horizon ex- traction, the coarse-grid slopes will help to find the corresponding reflections and correct the horizon extraction. In addition, the multigrid correlations or slopes computed by dynamic warping can also assist in computing phase-consistent horizons.

TBA Houzhu Zhang,

TBA Sergey Fomel,

Graduate Student Posters

Uncertainty Quantification of the Wave Equation with Radiation Boundary Conditions Brian Citty, Department of Mathematics SMU

We will present an uncertainty quantification analysis of models which use a Dirichlet to Neumann operator to solve numerically an unbounded problem on a bounded domain. We will focus on the simple case of the 1D scalar wave equation. After selecting a suitable model of the random data, we will compare and contrast various schemes for solving the corresponding SPDE, and show how the distribution of solutions affects various functionals of interest.

A Kinetic Theory Approach to Pedestrian Motion

Daewa Kim, University of Houston

We present a kinetic approach theory approach to model pedestrian dynamics. This approach models the dynamics caused by the interactions of pedestrians with other pedestrians as well as with the boundary of the domain. Four factors are taken into account: (1) the pedestrian's goal (e.g., to reach an exit), (2) the desire to avoid collisions with the walls, (3) the tendency to look for less congested areas, and (4) the tendency to follow the stream unconsciously in a panic situation. Thanks to this approach, we simulate evacuation from a room under several different conditions.

On the sensitivity to model parameters in a filter stabilization technique for advection dominated advection-diffusion-reaction problems

Kayla Bicol (presenter) and Annalisa Quaini, Department of Mathematics University of Houston

We consider a filter stabilization technique with a deconvolution-based indicator function for the simulation of advectiondominated advection-diffusion-reaction (ADR) problems with under-refined meshes. The proposed technique has been previouslyapplied to the incompressible Navier-Stokes equations and has been successfully validated against experimental data. However, it was found that some key parameters in this approach have a strong impact on the solution. To better understand the role of these parameters, we consider ADR problems, which are simpler than incompressible flow problems. For the implementation of the filter stabilization technique to ADR problems we adopt a three-step algorithm that requires (i) the solution of the given problem on an under-refined mesh, (ii) the application of a filter to the computed solution, and (iii) a relaxation step. We compare our deconvolution-based approach to classical stabilization methods and test its sensitivity to model parameters on a 2D benchmark problem.

GPU-accelerated Bernstein-Bezier weight-adjusted discontinuous Galerkin methods for wave propagation in heterogeneous media

Kaihang Guo, Computational and Applied Mathematics (CAAM) Rice University

Efficient and accurate simulations of wave propagation are central toapplications in seismology. In practice, heterogeneities arise from the presence of different types of rock in the subsurface. Additionally, simulations over long time periods require high order approximation to avoid numerical dispersion and dissipation effects. The weight-adjusted discontinuous Galerkin (WADG) method delivers high order accuracy for arbitrary heterogeneous media. However, the cost of WADG method grows rapidly with the order of approximation. To reduce the computational complexity in high order methods, we propose a Bernstein-Bezier WADG method, which takes advantage of the sparse structure of matrices under the Bernstein-Bezier basis. Our method reduces the computational complexity from $O(N^6)$ to $O(N^4)$ in 3D and is highly parallelizable to implement on Graphics Processing Units.

Existence and Uniqueness of Solutions of the Equationsof Poroelectroelasticity Yu Hu, Department of Mathematics, SMU

We consider solutions of the equations of poroelectroelasticity, which are a combination of Maxwell's equations and the equations of poroelasticity. The uniqueness of the solution is shown first, under general assumptions about the coefficients and initial and boundary conditions. Then the existence of a solution of Maxwell's equations is derived from results for evolution equations. By combining the properties of the solution operator of the elasticity equations and boundedness of the solution operator of Maxwell's equations, and applying a modified Rothe's method, we show the existence of a solution. This work was supported by grant 1619969 from the NSF.

An Energy Based DG Method for Nonlinear Wave Equations Lu Zhang, SMU

In recent years, discontinuous Galerkin methods have proven to be an effective approach to developing highorder, energy-stable discretizations of time-domain wave propagation problems in complex geometry. Here we present a new method for constructing discontinuous Galerkin methods for wave equations in second order form. The weak form works directly with the Lagrangian of the system. By Noether's Theorem, given any symmetry of the Lagrangian, one can derive a conservation law on a given element. Our method is built on such conservation laws, with the weak form chosen so that the rate of change of the conserved quantity is determined by the flux through the element boundaries. This method has been applied to a variety of problem ranging from the simple scalar wave equation, linear elasticity to convective second order wave equation. In this work, we focus on its application to nonlinear second order wave equations. We will give the idea of our DG method and demonstrate optimal high-order convergence for some simple examples.

Understanding the Effect of Measurement Time on Drug Characterization Hope Murphy, Texas Christian University

In order to determine correct dosage of chemotherapy drugs, the effect of the drug must be properly quantified. There are two important values that characterize the effect of the drug: max is the maximum possible effect from a drug, and IC50 is the drug concentration where the effect diminishes by half. We use mathematical models to estimate how the values depend on measurement time and model choice. Improper choice of growth model is problematic and can lead to differences in predictions of treatment outcomes for patients. This work intends to understand how choice of model and measurement time affects the relative drug effect and causes the differences in predictions for the most effective dose of anticancer drug for a patient. This work determines the correct doses before trying those in patients to get the most effective therapeutic treatment.

A social mobility model approaches to the discrete time and stochastic epidemiology model. Ye Li, Department of Mathematics Texas Tech University.

We introduce an SIR model coupled to a social mobility model (SMM). We discretize by a forward Euler Method, and a mixed type Euler method (structured with both forward and backward Euler elements). We also structured a stochastic differential model for our social mobility model. We calculate basic reproduction number R0 using a next generation matrix method. When R0 < 1, there will be a disease-free equilibrium (DFE), and R0 < 1 implies DFE will be locally asymptotically stable, while R0 > 1 implies DFE is unstable. Then we consider the SIR model in influenza and plug our models, we compare with the influenza data from CDC/NIH on 2018.

Dynamics and Noise Performance of Periodically-stationary Pulses in Fiber Lasers

Vrushaly K. Shinglot and John Zweck(1), Curtis R. Menyuk(2), Yannan Shen(3), 1: University of Texas at Dallas; 2: California State University, Northridge; 3: University of Maryland, Baltimore County;

Fiber lasers operating in the dispersion-managed and similariton regimes generate periodically-stationary pulses that undergo large changes each round trip and whichform the basis of frequency combs for applications to time and frequency metrology. Using a lumped rather than an averaged system model, we develop an efficient computational method to determine periodically-stationary solutions. We assess the stability of these solutions and we quantify the noise performance of the system in terms of the variances of the central time and phase of the pulse, and the width of lines in the frequency comb. Specifically, we derive a formula for the comb linewidths in terms of the timing and phase variances, and we show that for fiber lasers operating in the similariton regime the minimum linewidths occur at a small negative value of round trip dispersion.

Photonic Crystal Fibers With A Twist

Austin Copeland, Department of Mathematics SMU

Light confinement can occur during the propagation through twisted photonic crystal fiber (a chiral fiber). In the absence of a twist, the modal profile is assumed known fromBloch theory. By use of averaging techniques applied to raytheory, we attempt to describe the optical path length leading to confinement. Alternatively, we also explore the problem using field theory described by the linear Schrodingerequation. We show that an increase in twist rate will resultin more confined modes.

Solution of a time fractional system of homogeneous KDV equations Sasan Mohyaddin, Mathematic Department SMU

New traveling wave solutions are established by using the modified extended tanh method for time fractional nonlinear systems. By employing the method, soliton solutions are obtained for different types of time fractional systems, such as time fractional two?component evolutionary systems of a homogenous KdV equation of order two and three. Based on a fractional complex transform and the properties of modified Riemann-Liouville derivative, both systems are reduced to ordinary differential equations. The exact solutions for these systems are plotted at different time levels.

Decoys and dilution: the impact of incompetent hosts on prevalence of Chagas disease Zahid MD Monday Hasan, University of Texas, Austin

Biodiversity is commonly believed to reduce risk of vector borne zoonoses. This study focuses on the effect of biodiversity, specifically on the effect of the decoy process (additional hosts distracting vectors from their focal host), on reducing infections of vector borne diseases in humans. Here, we consider a specific case of Chagas disease and try to observe the impact of the proximity of chickens, which are incompetent hosts for the parasite but serve as a preferred food source for vectors. We consider three cases as the distance between the two host populations varies: short (when farmers bring chickens inside the home to protect them from predators), intermediate (close enough for vectors with one host to detect the presence of the other host type), and far (separate enclosed buildings such as a home and henhouse). Our analysis shows that the presence of chickens reduces parasite prevalence in humans only at an intermediate distance and under the condition that the vector birth rate associated with chickens falls below a threshold value, which is relative to the vector birth rate associated with humans and inversely proportional to the infection rate among humans.

The Dynamics of a Nutrient-Phytoplankton-Zooplankton (NPZ) Model

J. Caleb Macdonald, Department of Mathematics University of Louisiana

NPZ Models are a popular ODE system used by both mathematicians and biologists to examine plankton population dynamics in the relative short term. This poster will give an overview of the dynamics these systems can display through analysis of an example model. Limitations of the model type will be discussed as well as likely future directions in plankton population modeling.

Surrogate Models for Discrete Fracture Networks using Graphs Jaime Lopez,

Reduced modeling is becoming increasingly necessary to yielding fast results for complicated, expensive models. Although many models rely on a similar modeling methodology for reduction, ie taking a matrix system and reducing it while maintaining of the same properties as the larger system, there are other avenues. A large-scale computational model for transport was developed at Los Alamos National Laboratory that uses discretized meshes for the forward solution. A reduced model was created using graph networks to simplify the problem. We propose to model quantities of interest, such as pressure at fracture intersections, by using the graph model. We propose to test our uncertainties against well-established results of the high- fidelity model as proof of trust for use in larger and unknown transport problems. Dynamics and Noise Performance of Periodically-stationary Pulses in Fiber Lasers Vrushaly K. Shinglot(1), John Zweck(1), Curtis R. Menyuk(2), Yannan Shen(3), 1: University of Texas at Dallas; 2: University of Maryland, Baltimore County; 3: California State University, Northridge

Fiber lasers operating in the dispersion-managed and similariton regimes generate periodically-stationary pulses that undergo large changes each round trip and which form the basis of frequency combs for applications to time and frequency metrology. Using a lumped rather than an averaged system model, we develop an efficient computational method to determine periodically- stationary solutions. We assess the stability of these solutions and we quantify the noise performance of the system in terms of the variances of the central time and phase of the pulse, and the width of lines in the frequency comb. Specifically, we derive a formula for the comb linewidths in terms of the timing and phase variances, and we show that for fiber lasers operating in the similariton regime the minimum linewidths occur at a small negative value of round trip dispersion.

Parameter Sensitivity Analysis of Dynamics of Ovarian Tumor Growth Model Md. Shah Alam, Texas Tech University, md-shah-alam.md-shah-alam@ttu.edu

Cancer is the proliferation (growth) of malignant cells, that grow improperly, interrupt normal tissue structure and function. Ovarian cancer is when abnormal cells in the ovary begin to multiply out of control and form a tumor. If left untreated, the tumor can spread to other parts of the body. We present a simple mathematical model of Delay Differential Equations to describe the dynamics of ovarian tumor growth. A mathematical expression called Droop's cell quota model which governs the tumor growth, where cell qouta represents the intracellular concentration of necessary nutrients provided through blood supply. This mathematical model can be used for both On and Offtreatment but here we are interested in parameter sensitivity analysis of that model using Latin Hypercube Sampling (LHS) and Partial Rank Correlation Coefficient (PRCC) subject to On anti-angiogenesis treatment. We simulate the Tumor volume(y), Cell nutrient density(Q) and the Maximum size of tumor(ymax) using LHS-PRCC to find out which parameters are important for the model and for the treatment. We are also interested in comparisons of simulated results to the preclinical data. To get the analytical solution and corresponding result we use MATLAB.

Undergraduate student contributions are listed in different document.