Second Annual

LA-TX Undergraduate Mathematics Conference

in conjunction with the first annual TX-LA SIAM sectional meeting
at Louisiana State University, Baton Rouge
Friday, October 5, 2018

About the conference

The LA-TX Undergraduate Mathematics Conference is a cooperation among Louisiana State University, the University of Houston, and Texas A&M University. The inaugural conference was held in November of 2017 at the University of Houston. This year, it is incorporated with the SIAM TX-LA sectional meeting and invites participation from all other universities in Louisiana, Texas, and elsewhere.

The undergraduate conference will take place at a single forum so that all participants will be able to listen to all presentations by their peers. The poster session will include all of the projects that are presented orally, allowing people to inquire in more depth about the topics.

Support

The LA-TX Undergraduate Mathematics Conference is supported financially by the LSU Mathematics Department, by the National Science Foundation, by SIAM, and through registration fees of the participants; and facilities are granted by Louisiana State University.

We are grateful for logistical support from LSU math staff Soula O'Bannon, Kathy Ulkins, and Charlotte Senn; and LSU students Kacie Shepeard, Dillon Deshotel, Dakota Babin, Ricky Tran, and Nathan Anderson.

Organizers

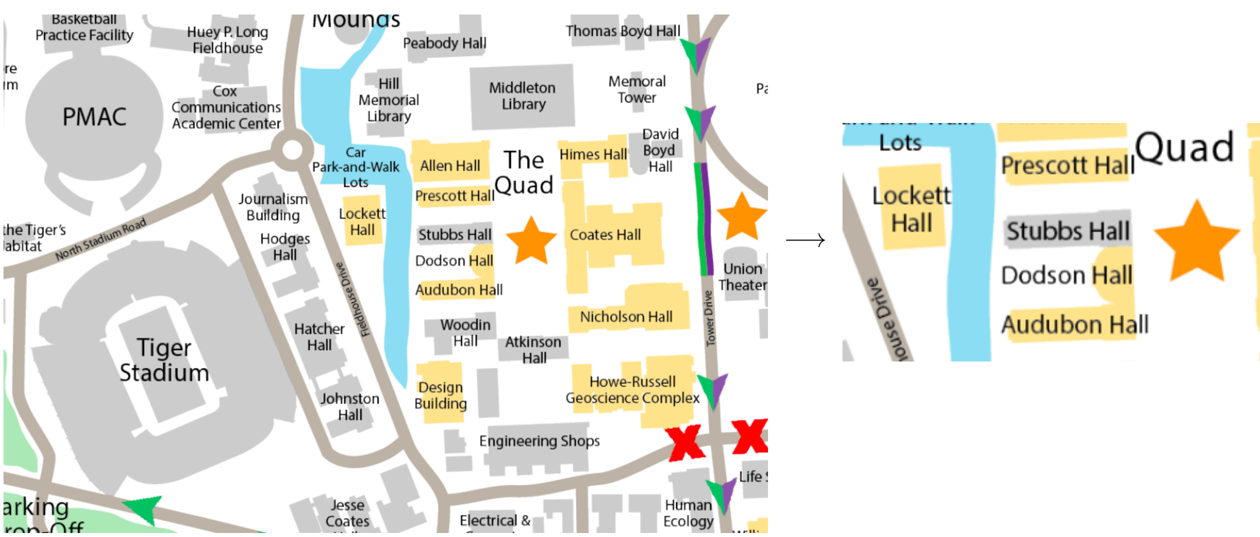
LSU: Robert Lipton and Stephen Shipman
UH: William Ott, Daniel Onofrei, and Andrew Török
Texas A&M: Michael Anshelevich
SIAM and Chevron: Anusha Sekar

Participants

Presenters are undergraduate students from Texas A&M, U Houston, LSU, Ole Miss, Tulane, UT Rio Grande Valley, SMU, GA Tech, Wake Forest, Southern Miss; and high school students from Louisiana.
Special lecturer is Prof. Ricardo Cortez from the Mathematics Department at Tulane University.
Audience will include mathematicians from academia and industry from TX, LA, and elsewhere.

Forum

Most of the conference will be held in the Auditorium of Dodson Hall. Lunch and the poster session will be held in Lockett Hall, which is the mathematics building of LSU.



Schedule

8:00	Registration and welcome in Dodson Hall Words from the Chair of LSU Mathematics, Dr. Oliver Dasbach
8:30	On persistence of superoscillations for the Schrödinger equation with time-dependent quadratic Hamiltonians Jose Palacio, UT Rio Grande Valley
8:50	Classification of images from noisy observations with a coupled inversion-classification neural network Daniel Flores and Cayce Rimland, University of Houston
9:10	Double Bubble Problem in the Taxicab Metric Rory O'Dwyer, Texas A&M University
9:30	Mathematical Measures of Fairness in Legislative Districting Spencer (Brady) Gales, Wake Forest University; and Bryson Kagy, Georgia Institute of Technology
9:50	Guest Lecture: Ricardo Cortez, Tulane University
10:30	Coffee break
10:50	Scattering Cancellation Using Dipole Arrays Charlene Woelfel, University of Houston
11:10	Encryption via Cyclic Permutations Ali Marzoughi, Baton Rouge Magnet High School and LSU Math Circle
11:30	Monitoring and Analyzing Vital Signs for Athletic Programs Aliza Bista, Jun Lin, and Michael Sheppard, Louisiana State University
11:50	On a Generalization of the Fibonacci Sequence Victoria Robinson, University of Mississippi
12:10	Solving Differential Equations Numerically Jay Standridge, Texas A&M University
12:30	Lunch in the third-floor lounge of Lockett Hall
1:40	Squaring an Oriented Graph and K-color Squaring Harris Cobb, Texas A&M University
2:00	Stochastic Numerical Solutions Stephanie Flores, UT Rio Grande Valley
2:20	Particle Tracking for Live Cell Data Riley Juenemann, Tulane University
2:40	Breaking barriers in the analysis of generation means and inference of genetic architecture Andrew Armstong, Texas A&M University
3:00	Modeling Networks of Evolving Populations Sean Elliott, Southern Methodist University
3:20	Coffee break
3:40	Computational Methods in the Study of Interdependent Graphs Carter Koehler, Southern Methodist University
4:00	A Predator-Prey Model with Hunting Cooperation among Predators and Allee Effects in Prey Aaditya Kharel, The University of Southern Mississippi
4:20	Discovering Temporal Correlations in the Evolution of Discrete Traits Nathan Anderson, Texas A&M University
4:40	Dynamic Coverage of Sensor Networks Samad Ahmed, University of Houston
5:10	Panel Discussion, Dodson Hall Panelists: Scott Baldridge (LSU Math); Tan Bui (UT Austin ICES); William Ott (UH Math); Erin Mire (Chevron North America Exploration and Production Company)
6:00	Dinner on your own
7:30	**POSTER SESSION: Two hours with refreshments** Second floor of Lockett Hall
9:30	End

Abstracts

Plenary Presentation

Computational Models of Microscopic Fluid Flows with Biological Applications

Prof. Ricardo Cortez, Mathematics Department, Tulane University

The term Biofluid dynamics refers to the motion of fluids generated in biological and biomedical phenomena. These include blood flow in capillaries, the flows surrounding moving cells, bacteria, spermatozoa, and more. Many of these phenomena involve a flexible, elastic body exerting forces to the fluid in order to propel itself or generate flows that are advantageous in some way. These phenomena are of interest to scientists who study how the fluid environment and the forces generated on the elastic bodies combine to produce organism behaviors observed in the laboratory. I will present the main ideas behind a computational model called "the method of regularized Stokeslets" that is widely used to study microscopic flows generated by flagella (like those attached to bacteria and sperm cells). The presentation will be based on vector calculus and introductory partial differential equations. I will show simulations of flagellar motions that aim to understand the effect of asymmetry in the flagellar beat patterns as well as interactions with a nearby surface, which is important in fertilization.

Oral and Poster Presentations

On persistence of superoscillations for the Schrödnger equation with time-dependent quadratic Hamiltonians

Jose Palacio and Elijah Hight, University of Texas Rio Grande Valley
Advisors: Tamer Oraby and Erwin Suazo

The main purpose of this project is to study the area of wave phenomena that appear naturally in various areas of science such as quantum physics, biology, theoretical optics, nonlinear fluid dynamics, and turbulence. After an exhaustive investigation and revision of several publications in the same area we find different models which explain the movement of quantum oscillators buffered to Schrdinger equations which are time dependent with variable quadratic Hamiltonians. The main objective in this project was to solve in an explicit manner the equations that were left unresolved in previous works. Collaborators have been able to construct explicit solutions for a Schrdinger equation with variable coefficients (VCNLS). Depending on the elections of the coefficients equation, the applications vary in very specific problems as Bose-Einstein condensates, dispersion-managed optical fibers and soliton lasers, pulse dynamics in the dispersion-managed fibers. The approach depends in finding a general solution for a nonlinear system of ordinary differential equations associated to the coefficients of the equation. This system is known as Riccati-Ermakov system. With this system of solutions our goal is to build explicit solutions for VCNLS using generalized Fourier transform. These techniques will provide six parameters. This will provide an interpretation of the parameters related to the dynamics of the central axis of symmetry of the traveling wave solution. Numerical simulations of more general problems are employed where analytical techniques fail.

Classification of images from noisy observations with a coupled inversion-classification neural network

Daniel Flores and Cayce Rimland, University of Houston
Advisor: Lars Ruthotto, Emory University

In this paper, we present a neural network architecture to classify noisy observation data. We simulate noisy observations by passing images through an ill-posed forward operator, then add Gaussian white noise. The challenge arises from the fact that the images we wish to classify are only indirectly observable. In certain fields such as computed tomography, these types of noisy observations need to be reconstructed in some way before they can be classified by a neural network. Our proposed network couples these tasks together into a combined inversion-classification network which reconstructs and classifies images at once. We accomplish this by unrolling the reconstruction process into a deep neural network by treating some of the reconstruction parameters as trainable weights, this is known as a variational neural network, we then feed the output of this network to a deep convolutional neural network. this combined inversion-classification network may be used when what is of interest is not the reconstruction in and of itself, but the classification based on the reconstruction.

Double Bubble Problem in the Taxicab Metric
Rory O'Dwyer, Texas A&M University
Advisor: Eviatar Procaccia, Texas A&M

We obtain the curve in R^2 under the L_1 metric that minimizes joint perimeter for two contained fixed areas. In doing so, we take advantage of properties of the already known solution for the case of 1 area to reduce the body of eligible curves. The remaining candidates are then further processed using the Karush-Kuhn-Tucker variation on the Lagrange Method, from which we obtain the unique minimizer.

Mathematical Measures of Fairness in Legislative Districting
Spencer (Brady) Gales, Wake Forest University; **Bryson Kagy**, Georgia Institute of Technology
Advisors: David Offner and Jessica De Silva, Carnegie Mellon University

Gerrymandering is the term used to describe the drawing of legislative districts to favor one party over another. Recently, mathematicians have tried to develop mathematical tools to decide if legislative districts are gerrymandered, and define fair methods of districting. For example, Landau, Reid, and Yershov [A Fair Division Solution to the Problem of Redistricting, Social Choice and Welfare, 2008] propose a protocol for districting based on a two-player fair-division process, where each player is entitled to draw the districts for a portion of the state. We call this the LRY protocol. Landau and Su [Fair Division and Redistricting, arXiv:1402.0862, 2014] propose a measure of the fairness of a districting called the geometric target. In this paper we prove that the number of districts a party can win under the LRY protocol can be at most two fewer than their geometric target, assuming no geometric constraints on the districts. This is the first quantitative bound of this type and we provide examples to prove this bound is tight. The main tools involved in the proof are identifying optimal strategies for each player in the protocol, and analyzing the number of districts they win using these strategies. We also show that the protocol on a state drawn with geometric constraints can produce an unbounded difference from the geometric target. Lastly, we explore ways to generalize the LRY protocol to m players, and define a similar protocol with lower bounds on number of victories

Scattering Cancellation Using Dipole Arrays
Charlene Woelfel and Neil Egarguin, University of Houston
Advisor: Daniel Onofrei

We present a strategy for cancelling the scattered far field of a dipole interrogating a perfect conducting plane. the interrogating dipole acts as a monostatic radar (*i.e.,* interrogator collocated with receiver) and for the scattering cancellation effect we are using a distribution of infinitesimal dipoles locate above the target. The goal is to determine the current feed required for the defending dipoles to cancel the scattered far field on a patch around the interrogating dipole while having a near-zero effect on the field elsewhere. The additional effect of very small fields outside a small patch around the interrogating dipole could lead to other important applications such as field focusing. We shall present some numerical simulations of this strategy applied to the scattering cancellation of the interrogator's magnetic far field. We begin with the specificx case of a vertical interrogating dipole, in which the cancellation was done using vertical dipoles of appropriate currents. We then consider the general case of an arbitrarily oriented interrogating dipole. In this setup, cancellation is achieved using dipoles oriented in three orthogonal directions with appropriate currents.

Encryption via Cyclic Permutations
Ali Marzoughi, Baton Rouge Magnet High School
Advisor: Noah Winslow, LSU

This project is about various different methods of using permutations when encrypting a message or text. The basic Permutation Cipher using cyclic permutation multiplication to generate a key for encryption is nice, but it's far from secure. This project goes over a few ways to enhance the Permutation Cipher using either alphabetic shifts or the permutation of bigrams and trigrams to get past cryptanalysis methods such as letter frequency and anagramming.

Monitoring and Analyzing Vital Signs for Athletic Programs
Aliza Bista, Jun Lin, and Michael Sheppard, Louisiana State University
Advisors: Peter Wolenski and Sudip Sinha, LSU

TigerDash is an in-house web-based data visualization tool designed for the Athletic Department at Louisiana State University. It offers a simple way to group players by sports and position, and graph the data depending on the selected variables. The data, exported from the Polar website, can be uploaded using Excel sheets or CSV files. The generated graphs can be exported as PDF files. It also features an automated report generation module.

On a Generalization of the Fibonacci Sequence
Victoria Robinson, University of Mississippi
Advisor: Maksym Derevyagin

Fibonacci numbers have fascinated mathematicians for centuries, and they continue to charm us with their beauty and their habit of occurring in surprising and unrelated places. One defines the Fibonacci sequence by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ for $n = 3, 4, 5, \ldots$ and the initial conditions $F_1 = 1$ and $F_2 = 1$. So one can easily verify that the Fibonacci sequence is 1, 1, 2, 3, 5, ... The goal of the present research is to study properties of a generalization of the Fibonacci sequence. Motivated by Marcia Edson and Omer Yayenie's research on a generalization of the Fibonacci sequence in 2009 [Edson, Marcia, and Omer Yayenie. *A New Generalization of Fibonacci Sequence and Extended Binet's Formula*, Integers 9.6 (2009): 639–659], a decision to study a generalization given by

$$f_n = f_{n-1} + (-1)^n f_{n-2}, \quad n = 2, 3, 4 \ldots$$

and the initial conditions $f_1 = 1$, $f_2 = 1$. We study this generalization through computer software to compare the classical Fibonacci sequence and the generalization, form conjectures based on observations of the classical Fibonacci sequence, and prove some relations using the Principle of Mathematical Induction to form theorems.

Solving Differential Equations Numerically
Jay Standridge, Texas A&M University
Advisor: Kamran Reihani, Texas A&M

The topic of differential equations is an ever growing one. However, as in many mathematical problems, numerical methods are extremely useful, if not necessary. In this talk, we will explore the use numerical methods to solve differential equations, how two polynomials can be considered orthogonal, and how we can use a set of orthogonal polynomials to approximate the solution to differential equations, as well as the efficacy and limits of doing so. In short, one is able to solve an arbitrary linear differential operator with appropriate initial and/or boundary values to 6-16 decimal places depending on the difficulty of the problem and how many polynomials are used in the solution.

Squaring an Oriented Graph and K-color Squaring
Harris Cobb, Texas A&M University
Advisor: Peter Johnson, Auburn University

It was conjectured that for a directed graph there exists a vertex such that the outdegree is at least doubled in the square of the graph. We discuss a generalization which would follow as a corollary, and relate squaring bipartite oriented graphs with k-color squaring.

Stochastic Numerical Solutions
Stephanie Flores, Olivarez-Vargas, Jose Palacio, Elijah Hight, UT Rio Grande Valley
Advisors: Tamer Oraby, Erwin Suazo, Jasang Yoon

The first proposed problem is to work on the following stochastic Burgers equation with variable coefficients $u_t - a(t)u_{xx} = c(t)uu_x - b(t)x + F(x,t)$. The stochastic term in this equation, $F(x,t) = \sigma x\dot{W}$, where W is a standard Brownian motion, is a result of adding white noise to the time dependent parameter b. Burgers equation is an important model appearing in gas dynamics, turbulence, heat conduction, nonlinear acoustics, plasma physics and cosmology. We study explicit solutions with multi-parameters using Riccati-Ermakov systems and their dynamics. We perform numerical simulations for the case $F(x,t)$ being a stochastic function and work on solving the stochastic Burgers equation and compare solutions to the exact solution. Recently, the forced Burgers equation has attracted interest after the work by Polyakov who considered a white noise forcing term $F(x,t)$ and which can be transformed to KardarParisiZhang (KPZ) equation to model surface growth in the case of constant coefficient. In addition to this work, we have also been working on solving diffusion- type equations with the Riccati system of equations and computing them numerically. Problems such as the Forwards/Backwards Kolmogorov, the Black-Scholes equation, and the Fokker-Planck equation can be solved in this manner.

Particle Tracking for Live Cell Data
Riley Juenemann, Tulane University
Advisors: Scott McKinley, Christine Payne

In contrast to in vitro particle tracking experiments, wherein there are great controls on particle and environmental homogeneity, live cell (in vivo) tracking features tremendous diversity in particle movement. In this work, we have developed a suite of "first-pass" statistical tools to categorize disparate types of particle trajectories. The data we used for this project was generated in the the lab of Prof. Christine Payne, using fluorescence microscopy in HeLa (model human) cells. Some particle paths were easily distinguishable as free diffusion, stuck diffusion, or directed transport, while other trajectories were difficult to categorize. Several of the more complex paths indicated the potential for tracking error. The tools we developed for the categorization process include the correlation between consecutive increments and effective diffusivity from a maximum likelihood estimation. The standard deviation for the major and minor axis and the creation of a parameterized path to represent a fictional moving anchor employed principal components analysis. This anchor estimation allowed the computation of effective velocity and the average distance the particle deviated from the anchor. Based on these data measures, K-means clustering was utilized to distinguish between free diffusion, stuck diffusion, directed transport, and tracker error. This automated categorization process proved to be successful on data simulated using stochastic differential equations and provided interesting results on the live cell data.

Breaking barriers in the analysis of generation means and inference of genetic architecture
Andrew Armstrong, Texas A&M University
Advisor: Heath Blackmon, Texas A&M

The mapping of genotype to phenotype has important implications for the evolution of traits. For instance, alleles that act in additive fashion can quickly increase in frequency through natural selection while alleles that act epistaticallyare less likely to fix in response to selection. Line Cross Analysis (LCA) is one way of inferring the genetic architecture of a trait. With line cross analysis, we perform a crossing experiment between two strains, populations, or even closely related species that have very different values of a phenotype being studied. These two lines are then crossed to make an F1 and various types of backcross lines. Each of the lines generated has different proportions of parental genomes, levels of heterozygosity, and values of the phenotype being studied. This variation allows us to investigate the degree to which the trait of interest is determined by different composite genetic effects. While we cannot identify the behavior of any specific gene (no sequencing is done in this approach), these composite genetic effects represent the net action of all genes that influence the trait of interest. The types of composite effects that can be studied are defined by the line crosses produced. With very simple crossing designs, we can only investigate the degree to which additive, dominance, and simple two-partner epistasis explain the variation we see in the trait. However, with more complex designs, we can partition additive and dominance effects among sex chromosomes and autosomes. If our crossing design includes reciprocal crosses, we can investigate the role of mitochondria and maternal effects. Finally, with the software we have developed, we can also investigate the role of genotype by environment and genotype by sex interactions.

Modeling Networks of Evolving Populations
Sean Elliott, Southern Methodist University
Advisor: Dominic Skinner, MIT

We build a mathematical model for evolution. Our model is based on the Fisher-Eigen process, a differential equation that models the evolution of a probability density function describing the distribution of a population over a phenotype space. This equation depends on the choice of a *fitness function*, which represents the likelihood of survival at each value in the phenotype space. Because the behavior of a population under this equation is difficult to predict mathematically, we design a simpler model that successfully captures the dynamics of the original. To formulate this simpler model, we first simulate the Fisher-Eigen process over a 2-dimensional fitness landscape using MATLAB. For our design, we assume that organisms can be split into groups that share similar phenotype traits, and then the interactions between these groups can be studied. We then create a system of ordinary differential equations over the network of these groups, and we show that our model finds the correct equilibrium solution. Our prototype can be replicated by others to simplify modeling of high-dimensional data and to predict behavior based on data taken at a given time-step.

Computational Methods in the Study of Interdependent Graphs
Carter Koehler, Southern Methodist University
Advisor: Chrysafis Vogiatzis, NCA&T

The *critical node detection* problem has stood as a challenging, yet important problem for graph theorists for decades, and much is written about different attempts to solve it exactly or heuristically. The problem is even more challenging in interdependent graphs, where interaction effects between different graphs must be considered. Here, we propose a method for estimating node criticality in an interdependent graph setting using a measure of centrality based on the maximum degree of an induced star centered at a node. Com- putational results reveal that while, in general, our method does not perform better than other criticality metrics, it does show improved performance in highly modal graphs. In this work, we also propose methods for randomly generating interdependent graphs with specific degree correlation. This can be of use in test cases based upon different formu- lations of bivariate extensions for common random graph degree distributions, including power-law and Poisson distributions. Though limited in scope, these methods can be used to study interdependent graphs easier, especially the effects of degree correlation between multiple graphs. Finally, we provide the codebase, built upon *networkx*, which handles in- terdependent graphs and implements all our proposed computational methods and others, proposed by researchers in the same field. *Funded by DHS REU at North Carolina Agricultural and Technical University.*

A Predator-Prey Model with Hunting Cooperation among Predators and Allee Effects in Prey
Aaditya Kharel, Michelle McCullum, Nick Burks, The University of Southern Mississippi
Advisors: Zhifu Xie and Huiqing Zhu

Social interactions between individuals are an integral part of life history traits for many species (Courchamp et al, 2008). It is important that the predators and prey in any ecosystem coexist such that the presence of either of the species is not detrimental to their coexistence. Rather, the presence of both predator and prey species should provide a check and balance mechanism for each species to coexist and maintain their population in the ecosystem towards stable equilibrium. The Allee effect plays a significant role in determining the population dynamics of predator and prey to create stable coexistence equilibrium. We proposed a dynamical system of predator-prey interaction model consisting of two ordinary differential equations (ODEs) that incorporated hunting cooperation among the predators into a model of Rao and Kang. We investigate if hunting cooperation causes any Allee effect phenomena in prey. This is particularly important because if Allee effect is present in a two-species interaction model, we can predict that prey population below certain threshold will not survive at all in any ecosystem due to Allee effect.

Discovering Temporal Correlations in the Evolution of Discrete Traits
Nathan Anderson, Texas A&M University
Advisor: Heath Blackmon, Texas A&M

One of the central questions of evolutionary biology is how traits interact with each other over the course of evolution. Evolutionary biologists frequently wish to know if two traits are correlated or share some functional relationship. For example, nocturnality should, in theory, be correlated with the presence of camouflage because there is much less selective pressure on camouflage for organisms that are active in the dark and a biologist would like to confirm this hypothesis with a statistical test for correlation in the evolution of those two traits. However, most of the methods available for understanding correlation or contingency in the evolution of discrete traits are based on detecting differences in the rate of transitions in one trait when it is associated with a specific state of the other trait. These methods lead to several known problems, such as an elevated false positive rate due to a low number of transitions affecting a large large area of the tree. Transitions located early in the phylogeny, or before a number of speciation events are especially troublesome. We solve this so-called problem of within clade pseudo replication by developing an approach that focuses on temporal correlations in the transitions of discrete traits. Our statistical approach can determine whether transitions in one trait lead to transitions in a second trait more quickly or slowly than would be expected under a null model where the two traits evolve independently. This approach works around the problems of few transitions by indirectly inferring a correlation in rates through a change in the mean time to a transition in one trait to a transition in the other.

Dynamic Coverage of Sensor Networks
Samad Ahmed, University of Houston
Advisor: William Ott

Sensor networks are one of the areas that are widely studied in the interdisciplinary intersection of applied mathematics and engineering. We attack the coverage problem of sensor networks using homological techniques and generalize the coverage problem by relaxing boundary conditions. We then provide a criterion for dynamic coverage and explore a game-theoretical approach.

Irreducibility of the Fermi Surface for Planar Periodic Graphs
Daniel Rockwell, Louisiana State University
Advisors: Stephen P. Shipman and Wei Li

The Fermi Surface of a periodic Schrödinger operator is the set of all complex wavevectors admitted by the operator for a given energy. For discrete (graph) operators, it is essentially the zero set of a polynomial in several variables. Under special circumstances, this polynomial is factorable, and this leads to multiple algebraic components (reducibility) of the Fermi Surface, which has important implications for the spectral theory of the graph. This project addresses a conjecture that all periodic planar graph operators have irreducible Fermi Surface, and the proof involves computer computations of Groebner bases.

The 100 Prisoner Problem
Molly Hand, Alfred M. Barbe High School, Lake Charles, LA and LSU Math Circle
Advisor: Steven Olsen, LSU and Math Circle, Inc.

The 100 prisoners problem is a well-known question in the area of probability. One hundred prisoners follow stipulations set by the prison director in an attempt to free themselves. I found strategies for two variations following setups established by the prison director and calculated the probabilities that the prisoners would be pardoned using these strategies.

Jason Curves (Cyclocycloids)
Robert Gibson and Max Bhuiyan, Baton Rouge Magnet High School and LSU Math Circle
Advisor: Steven Olsen, LSU and Math Circle, Inc.

This project is centered around the analysis and research of complex parametric hypocycloid curves (specifically the three armed variant). The initial purpose was to find general properties of said curves through the precise changing of 6 variables (within $\mathbb{Z}$) included within this variant. These curves, called "Jason Curves", hold characteristic patterns within their formation. If enough time was allotted the exact curve properties of every possible Jason Curve could be procedurally discovered.

Fermat's Little Theorem
Ali Marzoughi(1), Niles Babin(2), Andrew Trepagnier(3), LSU Math Circle and 1: Baton Rouge Magnet High School; 2: University High School, Baton Rouge; 3: Catholic High of Pointe Coupee, New Roads, LA.
Advisor: Steven Olsen, LSU and Math Circle, Inc.

This project is about Fermat?s Little Theorem, a proof of it, and how we can use it to solve a difficult math problem. The project explains who Pierre de Fermat is and how he came about this theorem, as well as a simple proof by induction to help explain the theorem. Then the theorem is applied to 2005 USAMO Problem Number 5 and is used to solve it. The project is then concluded with more real-world applications of the theorem and future work that could be done with it.