Electromagnetic waves in a homogeneous medium

Let us consider traveling-wave solutions to the Maxwell equations in a homogeneous medium without sources:

$$\nabla \times E = -\frac{1}{c}\frac{\partial}{\partial t}\mu H \;, \quad \nabla \cdot \varepsilon E = 0$$
$$\nabla \times H = \frac{1}{c}\frac{\partial}{\partial t}\varepsilon E \;, \quad \nabla \cdot \mu H = 0$$

First, let E and H be harmonic in time:

$$E = E(\bar{x})e^{i\omega t},$$
$$H = H(\bar{x})e^{i\omega t}.$$

This form yields the harmonic Maxwell equations for $E(\bar{x})$, $H(\bar{x})$:

$$\nabla \times E = -\frac{i\omega}{c}\mu H,$$
$$\nabla \times H = \frac{i\omega}{c}\varepsilon E.$$

Now let $E(\bar{x}) = E_0 e^{i(k_1 x + k_2 y + k_3 z)}$, $H(\bar{x}) = H_0 e^{i(k_1 x + k_2 y + k_3 z)}$, so that E and H are travelling waves, as functions of time, in the direction of $-k = -\langle k_1, k_2, k_3 \rangle$:

$$(*) \qquad E = E_0 e^{i(k_1 x + k_2 y + k_3 z + \omega t)},$$
$$H = H_0 e^{i(k_1 x + k_2 y + k_3 z + \omega t)},$$

(E_0 and H_0 are constant vectors.)

The harmonic Maxwell equations, with the given spatial form, yield

$$(+) \qquad ik \times E_0 = -\frac{i\omega}{c}\mu H_0$$
$$ik \times H_0 = \frac{i\omega}{c}\varepsilon E_0$$

Solving for E, we obtain

$$k \times (k \times E_0) = -\left(\frac{\omega}{c}\right)^2 \varepsilon E_0$$

This tells us that E is perpendicular to k. More precisely,

$$k \times (k \times E_0) = -|k|^2 E_{0\perp} = -\left(\frac{\omega}{c}\right)^2 \varepsilon E_0,$$

where $E_{0\perp}$ is the component of E_0 orthogonal to k.
The second equality says that E_0 is a multiple of $E_{0\perp}$, so that $E_{0\parallel} = 0$, i.e., $E_0 = E_{0\perp}$. Thus we obtain

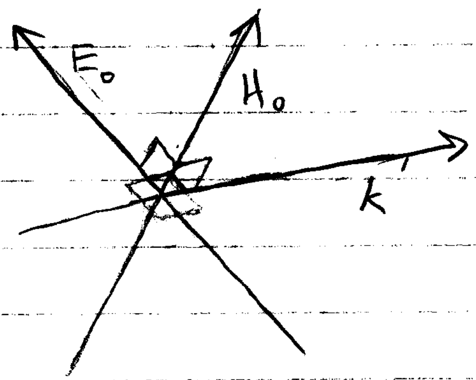
$$|k| = \sqrt{\varepsilon\mu}\,\frac{\omega}{c}.$$

Now, the speed of propagation of the waves (*) is equal to

$$v = \frac{\omega}{|k|} = \frac{c}{\sqrt{\varepsilon\mu}}.$$

If $\varepsilon = 1$, $\mu = 1$, as in a vacuum, then $v = c$.

Similar analysis applies to H_0. In addition,
(1) tells us that E and H are orthogonal to each other.
Thus, E, H, and k form an orthogonal system:



EM waves in a periodic medium: