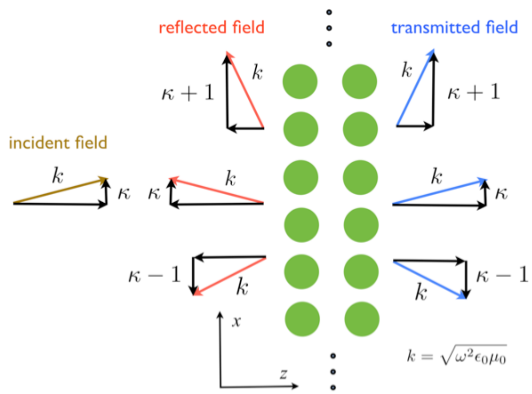

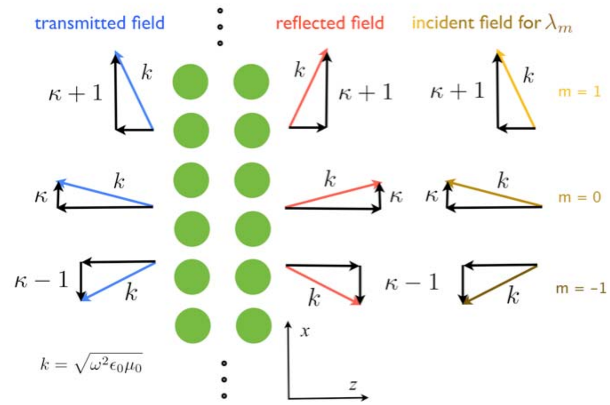

MODIFICATION OF ENERGY TRANSMISSION THROUGH PERIODIC ROD STRUCTURES

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Abstract

An algorithm is developed by which one can manipulate the coefficients of transmission, with the aim of controlling the amount, localization, and phase of the transmitted field. The algorithm is based on the variational calculus of the transmission coefficients (the objective functionals) as functions of the dielectric coefficients, centers, and radii of the rods. We use the "adjoint method", in which the variational gradient of the coefficient of each transmitted plane wave is represented by an adjoint problem. We present two examples in which we utilize the algorithm to modify a slab consisting of a periodic array of dielectric rods in air such that a desired transmission is obtained.

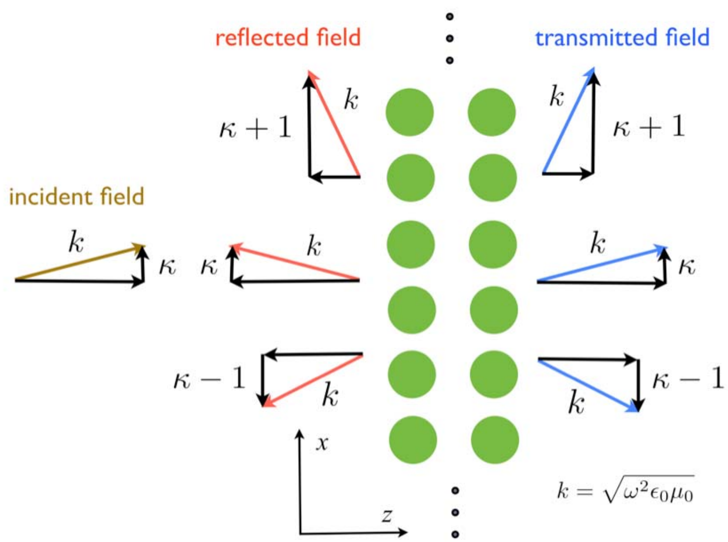
This project builds on the work of R. Lipton, S. Shipman, and S. Venakides (SPIE 2003)

SCATTERING PROBLEM RECAP

The scattering problem in strong form: Find a field $u(x,z)$ such that

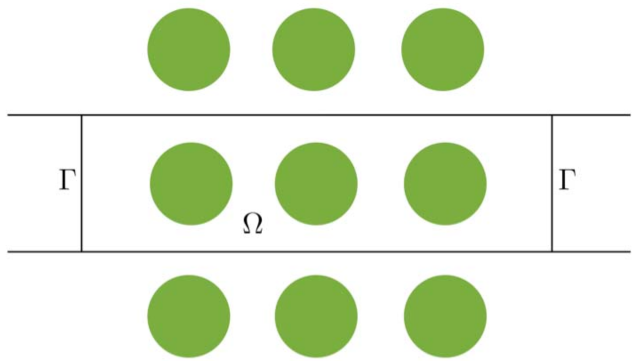
$$\begin{cases} \nabla^2 u(x,z) + \varepsilon\mu\omega^2 u(x,z) = 0 \quad \text{away from } \partial D \text{ (Helmholtz equation)} \\ \left.\begin{array}{l} u_{\text{int}} = u_{\text{ext}} \\ \mu_{\text{ext}}\partial_n u_{\text{int}} = \mu_{\text{int}}\partial_n u_{\text{ext}} \end{array}\right\} \quad \text{on } \partial D \text{ (conditions on the rod boundaries)} \\ u(x,z) = \tilde{u}(x,z)e^{i\kappa x}, \quad \tilde{u}\ 2\pi\text{-periodic in } x. \\ u = u^{\text{incident}} + u^{\text{scattered}}, \quad \text{with } u^{\text{scattered}} \text{ outgoing} \end{cases}$$

SOLUTION TO THE SCATTERING PROBLEM



General Solution:

$$u(x,z) = \begin{cases} e^{i(\kappa x + \eta_0 z)} + \sum_{m \in \mathbb{Z}} r_m e^{-i\eta_m z} e^{i(m+\kappa)x} & \text{to the left} \\ \sum_{m \in \mathbb{Z}} t_m e^{i\eta_m z} e^{i(m+\kappa)x} & \text{to the right} \end{cases}$$



The scattering problem in weak form: Find $u(x,z)$ such that

$$\begin{cases} \displaystyle\int_\Omega \left(-\mu^{-1}\nabla u \cdot \nabla\overline{\lambda} + \varepsilon\omega^2 u\overline{\lambda}\right) dA + \int_\Gamma \mu^{-1}\overline{\lambda}\partial_n u \, ds \, = \, 0 \\ \qquad\qquad \text{for all pseudoperiodic } \lambda \\ u \sim e^{i\eta_0 z}e^{i\kappa x} + \sum_{m=m_1}^{m_2} r_m e^{-i\eta_m z} e^{i(m+\kappa)x} \qquad (z \to -\infty) \\ u \sim \sum_{m=m_1}^{m_2} t_m e^{i\eta_m z} e^{i(m+\kappa)x} \qquad (z \to \infty) \end{cases}$$

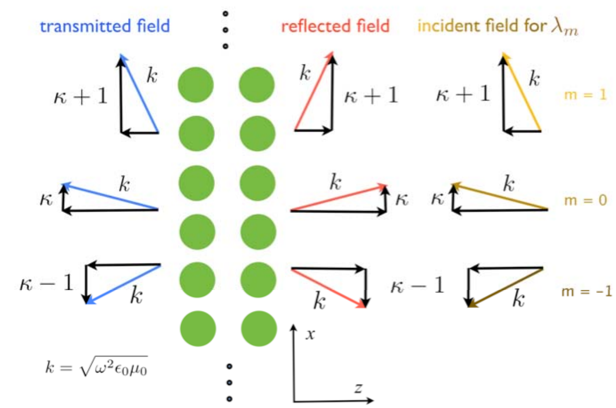
where $\quad \eta_m^2 + (m+\kappa)^2 = \varepsilon_0\mu_0\omega^2.$

VARIATIONAL CALCULUS

$$\varepsilon, \mu \longrightarrow u \longrightarrow \{t_m\}$$

$$\varepsilon + \hat{\varepsilon}, \mu + \hat{\mu} \longrightarrow u + \hat{u} \longrightarrow \{t_m + \hat{t}_m\}$$

THE ADJOINT PROBLEM



Find a pseudo-periodic function $\lambda_{m'}$ such that

$$\begin{cases} \int_\Omega \left(-\mu^{-1}\nabla\phi\cdot\nabla\overline{\lambda}_{m'} + \varepsilon\omega^2\phi\overline{\lambda}_{m'}\right) dA + \int_\Gamma \mu^{-1}\phi\partial_n\overline{\lambda}_{m'}\, ds = 0 \\ \qquad\qquad \text{for all pseudoperiodic } \phi \\ \lambda_{m'} \sim \sum_{m=m_1}^{m_2} \tau_m e^{i\eta_m z} e^{i(m+\kappa)x} \qquad\qquad \text{(left)} \\ \lambda_{m'} \sim t_{m'} e^{i\eta_{m'} z} e^{i(m'+\kappa)x} + \sum_{m=m_1}^{m_2} \rho_m e^{-i\eta_m z} e^{i(m+\kappa)x} \quad \text{(right)} \end{cases}$$

where $\eta_m^2 + (m+\kappa)^2 = \varepsilon_0\mu_0\omega^2$.

The ρ_m are the coefficients of the reflected field on the right, and the τ_m are the coefficients of the transmitted field to the left.

MODIFYING TRANSMISSION

$$\frac{\partial t_m}{\partial r_j} = \frac{\mu_{ext} i}{4\pi\eta_m} \int_{\partial D_j} [(\mu_j^{-1} - \mu_{ext}^{-1}) \nabla\psi \cdot \nabla\overline{\lambda}_m - (\varepsilon_j - \varepsilon_{ext})\omega^2 \psi \overline{\lambda_m}] ds$$

$$\frac{\partial t_m}{\partial x_j} = \frac{\mu_{ext} i}{4\pi\eta_m} \int_{\partial D_j} [(\mu_j^{-1} - \mu_{ext}^{-1}) \nabla\psi \cdot \nabla\overline{\lambda_m} - (\varepsilon_j - \varepsilon_{ext})\omega^2 \psi \overline{\lambda_m}] dx$$

$$\frac{\partial t_m}{\partial y_j} = \frac{\mu_{ext} i}{4\pi\eta_m} \int_{\partial D_j} [(\mu_j^{-1} - \mu_{ext}^{-1}) \nabla\psi \cdot \nabla\overline{\lambda}_m - (\varepsilon_j - \varepsilon_{ext})\omega^2 \psi \overline{\lambda_m}] dy$$

$$\frac{\partial t_m}{\partial \varepsilon_j} = \frac{\mu_{ext} i}{4\pi\eta_m} (-\omega^2 \int_{\partial D_j} \psi \overline{\lambda}_m dA)$$

THE JACOBIAN MATRIX

$$A = \begin{bmatrix}
\frac{\partial \Re t_{m_1}}{\partial x_1} & \frac{\partial \Re t_{m_1}}{\partial y_1} & \frac{\partial \Re t_{m_1}}{\partial r_1} & \frac{\partial \Re t_{m_1}}{\partial x_2} & \cdots & \cdots & \frac{\partial \Re t_{m_1}}{\partial r_n} \\
\frac{\partial \Im t_{m_1}}{\partial x_1} & & & & & & \frac{\partial \Im t_{m_1}}{\partial r_n} \\
\frac{\partial \Re t_{m}}{\partial x_1} & & & & & & \cdots \\
\frac{\partial \Im t_{m}}{\partial x_1} & & & & & & \cdots \\
\cdots & & & & & & \cdots \\
\cdots & & & & & & \cdots \\
\frac{\partial \Re t_{m_2}}{\partial x_1} & \cdots & & & & \cdots & \frac{\partial \Re t_{m_2}}{\partial x_n} \\
\frac{\partial \Im t_{m_2}}{\partial x_1} & \cdots & & & & \cdots & \frac{\partial \Im t_{m_2}}{\partial x_n}
\end{bmatrix}$$

A is M x N where

$$M = 2 \cdot (\#\text{ of propagating harmonics})$$

$$N = 3 \cdot (\#\text{ of rods})$$

CHANGE IN PARAMETERS

$$A\begin{bmatrix}\Delta x_1\\ \Delta y_1\\ \Delta r_1\\ \dots\\ \Delta r_n\end{bmatrix}=\vec{b} \longrightarrow AA^{\mathrm{T}}y=\vec{b} \longrightarrow \begin{bmatrix}\Delta x_1\\ \Delta y_1\\ \Delta r_1\\ \dots\\ \Delta r_n\end{bmatrix}=A^{T}y$$

The vector $\vec{b}=[\Delta\Re\, t_{m_1},\ldots,\Delta\Im\, t_{m_2}]$ contains the desired directions in which the transmission coefficients are to be moved, and the vector $[\Delta x_1,\Delta y_1,\Delta r_1,\ldots,\Delta x_n,\Delta y_n,\Delta r_n]^t$ contains the corresponding variations of the structural parameters.

THE ALGORITHM

1. Specify a rod structure whose transmission coefficients t_m we would like to modify.

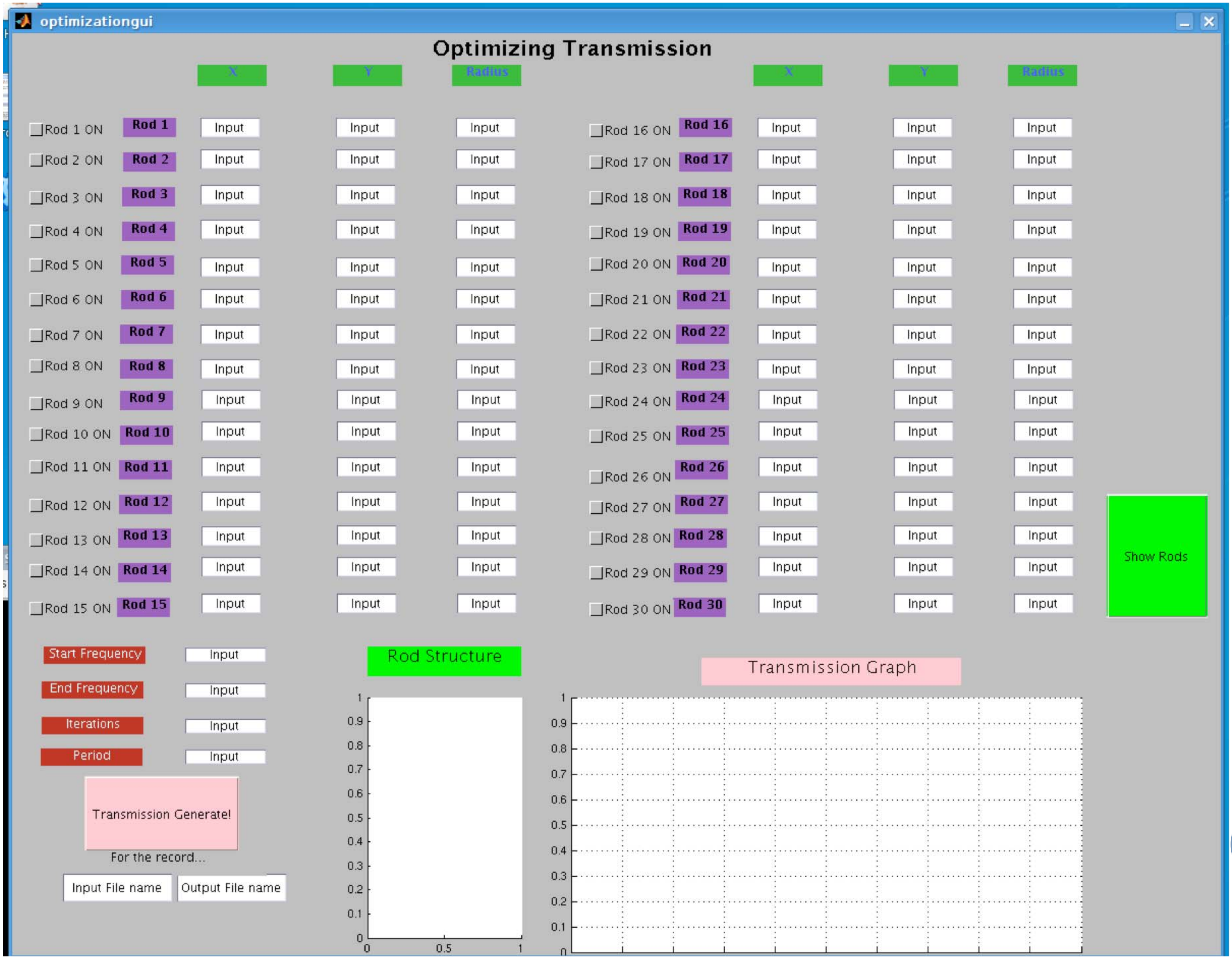
3. Find gradients of complex transmission coefficients with respect to structural parameters (rod centers and radii).

4. Choose a vector $\vec{b}$ of desired changes in t_m.

5. Solve matrix equation for the vector Δp such

   that $A\Delta p = \vec{b}$.

OUR GRAPHICAL USER INTERFACE



SIMULATIONS

Expanding the Modulus of Transmission Coefficients

For the first simulation, we take the pyramidal rod structure and attempt to manipulate the single propagating harmonic by steadily increasing the modulus while holding the phase constant. Using the previously described algorithm, for each iteration we aim to find a structure which will increase the modulus of the sole propagating mode t_0 by adding a small portion of the propagating mode to itself to create an outwardly dilating sequence.

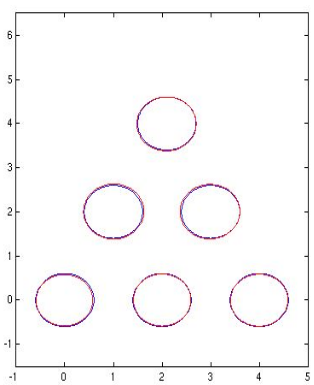
$$t_m^{\oplus} = t_m + \delta \cdot t_m = \left(\Re(t_m) + \delta \cdot \Re(t_m), \Im(t_m) + \delta \cdot \Im(t_m) \right)$$

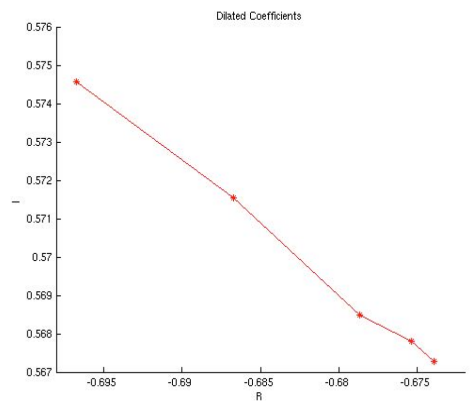
where t_m is the dilated mode we seek to achieve at each step and δ is the dilating factor.

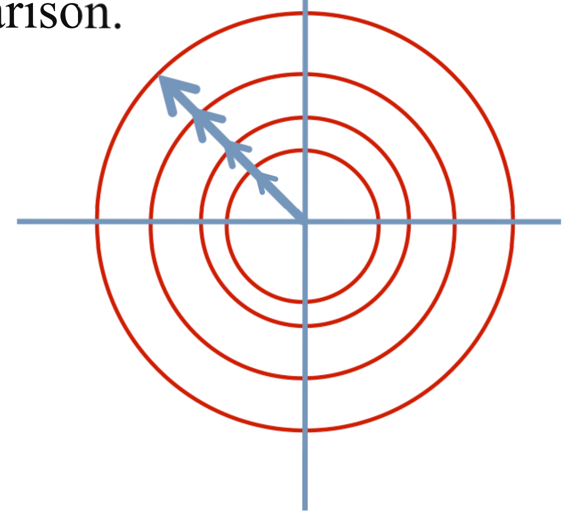
SIMULATIONS

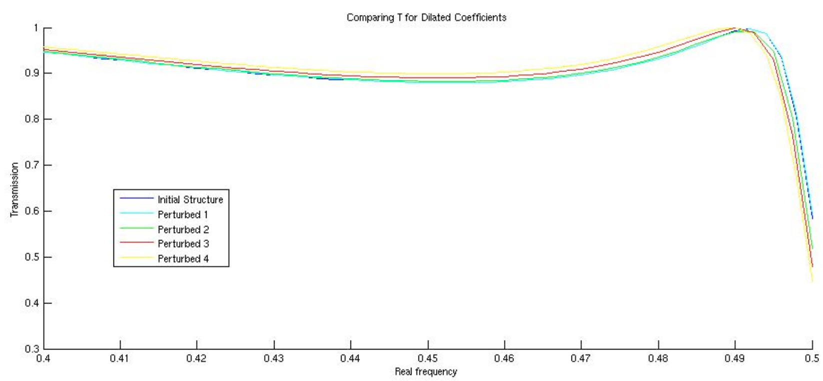
$$t_m^{\oplus} = t_m + \delta \cdot t_m = \left(\Re(t_m) + \delta \cdot \Re(t_m), \Im(t_m) + \delta \cdot \Im(t_m)\right)$$

We then compare the initial periodic structure and several perturbations with their corresponding complex transmission coefficients. Each structure's transmission graph is also plotted for comparison.









For δ less than five thousandths, the program dilated the propagating mode with little to no change in the angle θ.

SIMULATIONS

Changing the Phase

For the next simulation, we construct a period in the shape of a square lattice and attempt to change the phase of a single propagating harmonic while holding the modulus constant.

For each iteration, we now choose to add a small portion of $t_m^{\perp}$ to rotate the coefficients of the harmonic while maintaining a constant modulus.

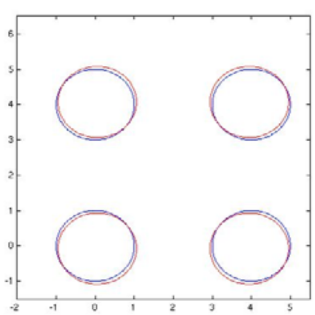
$$t_m^{\sigma} = t_m + \rho \cdot t_m^{\perp} = t_m + i(\rho \cdot t_m) = \left(\Re(t_m) - \rho \cdot \Im(t_m), \Im(t_m) + \rho \cdot \Re(t_m)\right)$$

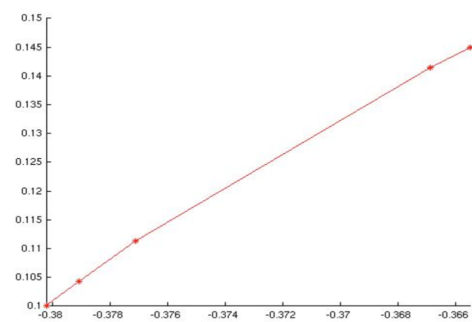
where $t_m^{\boldsymbol{\sigma}}$ is the rotated mode we seek for each iteration and ρ is the rotating factor.

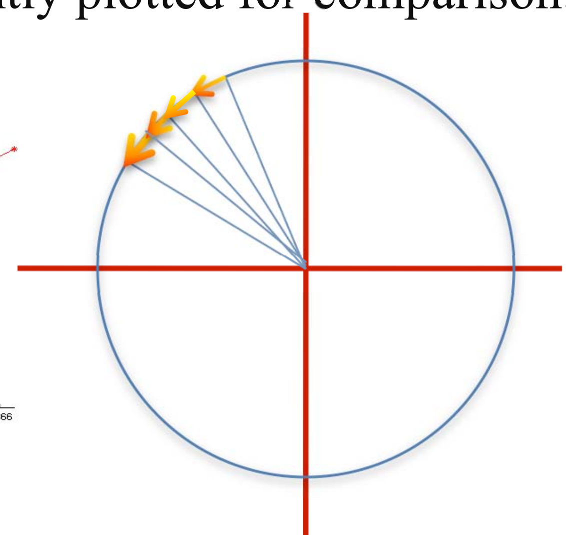
SIMULATIONS

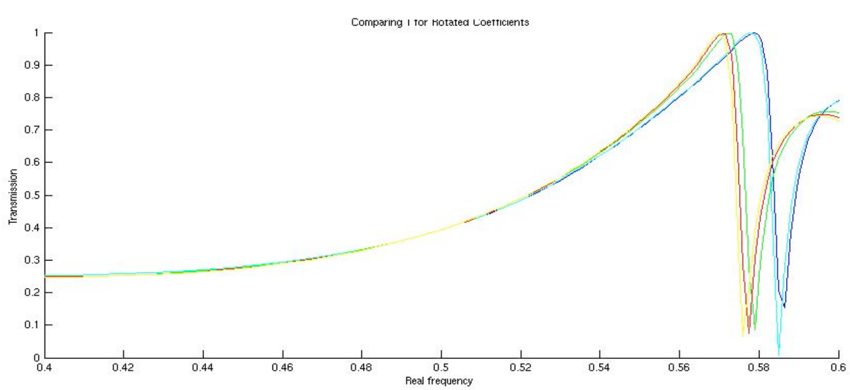
$$t_m^{\sigma} = t_m + \rho \cdot t_m^{\perp} = t_m + i(\rho \cdot t_m) = \left(\Re(t_m) - \rho \cdot \Im(t_m), \Im(t_m) + \rho \cdot \Re(t_m)\right)$$

The initial structure (in blue) and several of its perturbations (last in red) are shown along with their corresponding complex transmission coefficients. Also, the structures' transmission graphs are adjacently plotted for comparison.



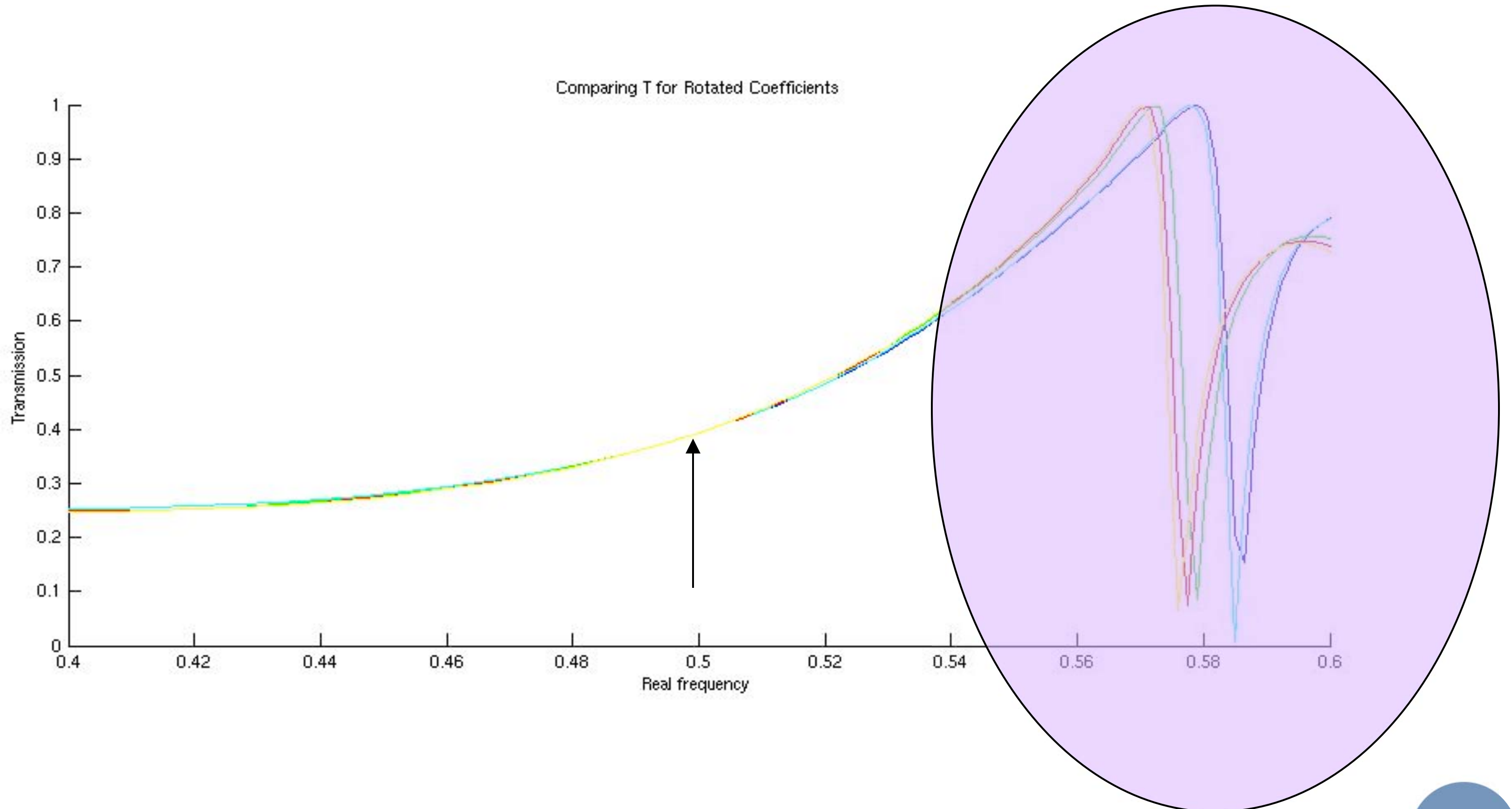






For ρ less than one thousandth, the algorithm successfully rotates the coefficients.

SIMULATIONS



Future Projects

Combining these results, one could manipulate the transmission of several propagating modes to localize the transmitted field to a small region.

$$u(x,z) = \sum_{n\in\mathbb{Z}} t_m e^{i\eta_m z} e^{i(m+\kappa)x}$$

So if t_m=t for every m, this becomes an infinite sum of cosines, and thus the energy will be localized for cos(Φ) when Φ = 0, 2π, …

This strategy can be used to optimize efficiency of solar power technologies.