7. Let \((f_2, g_2)\) be in the domain of \(A^*\). This means that the
map \((f_1, g_1) \mapsto (A(f_1), (g_2))\) is bounded, and thus the

Dresz theorem (cite of the density \(D^2 \oplus D' \subset D' \oplus L^2\)) provides

a unique pair \((f_2^*, g_2^*)\) such that \((A(f_1), (g_2)) = (f_2^*, (g_2^*))\).

This equality, written out in integrals form is

\[
\int w^2 g_1(w) \bar{f}(w) \, dw - \int w^2 f_1(w) g_2(w) \, dw
= \int w^2 f_1(w) \bar{f_2^*(w)} \, dw + \int g_1(w) \bar{g_2^*(w)} \, dw \quad \forall \left(\begin{array}{c} f_1 \\ g_1 \end{array}\right) \in D(A)
\]

By taking \(f_1 = 0\) and letting \(g_1\) range over all
functions in \(C_0^\infty\) (for example), we obtain \(A^*(\bar{f_2}) = \int w^2 f_2(w) \, dw \in L^2\)
so that \(f_2 \in L^2\). By taking \(g_1 = \bar{g_2}\), we obtain
\(f_2 \in D(A^*)\). But \(A(f_1) = g_2(w) \in L^2\)
and \(-g_2(w) = \bar{f_2^*(w)} \in D'\). Thus, \((f_2, g_2) \in D^2 \oplus D' = D(A^*)\)
so that \(\mathcal{S}(A^*) = \mathcal{S}(A)\). Moreover,

\[
A^*(f_2, g_2) = (f_2^*, g_2^*) = \left(\begin{array}{c} -g_2 \\ w^2 f_2 \end{array}\right) = -A \left(\begin{array}{c} f_2 \\ g_2 \end{array}\right).
\]

This means that \(A^* = -A\).
8. Let the projection $P$ be self-adjoint, and let $v \in \text{Null } P$ and $w \in \mathbb{H}$ be given. Then $(v, Pw) = (Pv, w) = (0, w) = 0$. Thus, $v \perp Pw$ and thus $\text{Null } P \perp \text{Range } P$.

Now suppose that $\text{Null } P \perp \text{Range } P$, and let $u, w \in \mathbb{H}$ be given. Since $P(I-P)w = (P-P^2)w = 0$, we \in \text{Null } P$.

We have $0 = ((I-P)w, Pw) = (w, Pv) - (Pw, Pw)$.

Similarly, $0 = (Pw, (I-P)v) = (Pw, v) - (Pw, Pw)$.

Thus, $(w, Pv) = (Pw, v)$.

Second point — easy enough.

9. Easy enough.

10. It is simple to show that $\frac{d^2}{dx} W[u, \tilde{u}_x] = (\tilde{u}_x)' u_x - (u_x)' \tilde{u}_x$, and this same expression and similarly equal to zero is obtained by multiplying $-5(\tilde{u}_x)' - \frac{k}{\lambda} u_x$ by $\tilde{u}_x$ and subtracting from the same equation with $\tilde{u}_x$ and $u_x$ switched. Thus, $W$ is constant.

Applying this result to a scalar field $u$ with $u(x) = e^{ikx + i\alpha} e^{-i\frac{k}{\lambda} x}$ for $x < L$ and $u(x) = 4e^{ikx}$ for $x > L$ gives $W[u, u] = u_x' - u_x = 2 \left[ -k + i\lambda r \right]$ for $x < L$ and $W[u, u] = -2k r^2$ for $x > L$. Since $W$ is constant, $W(L) = W(0)$ and so $|x|^2 + |t|^2 = 1$. 