

2012 F

Math 7386 Problem Set 3

8. Find the explicit solution of the problem

$$\begin{cases} u_t + (x^2 u)_x = 0 \\ u(x, 0) = f(x) \end{cases}$$

Use the method of characteristic curves to obtain $u(x, t)$.
[It is useful to check your result.]

Discuss the domain of validity of the solution.

This is a kind of equation of non-constant advection.

9. Find the solution $u(x, y)$ of

$$\begin{cases} u_y = (u_x)^3 \\ u(x, 0) = 2x^{3/2} \end{cases}$$

10. Solve $\begin{cases} u = x u_x + y u_y + \frac{1}{2}(u_x^2 + u_y^2) \\ u(x, 0) = \frac{1}{2}(1 - x^2) \end{cases}$

11. Find all solutions $u(x, y)$ of

$$\begin{cases} u_y = F(u_x) \\ u(x, 0) = h(x) \end{cases}$$