Math 2057-3,4 @ LSU
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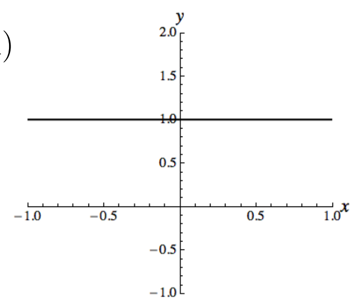
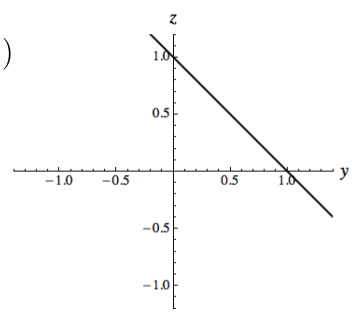
Exam 1
Tuesday, February 16, 2016

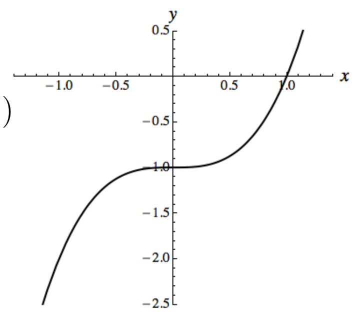
Each problem is worth eight points.
No calculators, books, notes, or other aiding materials are allowed.
Work the problems on the paper provided with this exam.

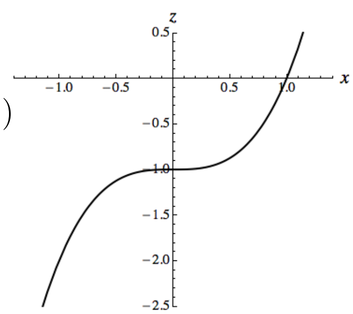
**1.** Match each of the descriptions with one of the figures.

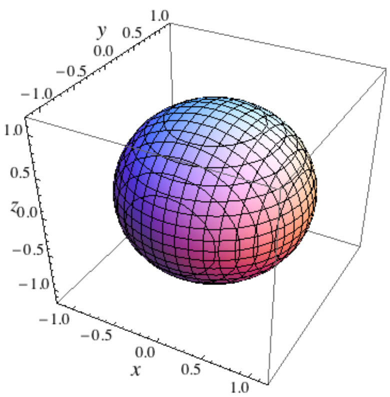
____**a.** Level set of $f(x, y) = x^3 - y$ for the value 1.
____**b.** Trace at $x = 1$ of the graph $z = f(x, y) = x^3 - y$.
____**c.** A level set of the function $f(x, y, z) = \sin\sqrt{x^2 + y^2 + z^2}$.
____**d.** Graph of the function $f(x, y) = \cos\sqrt{x^2 + y^2}$.

(A)



(B)



(C)



(D)



(E)
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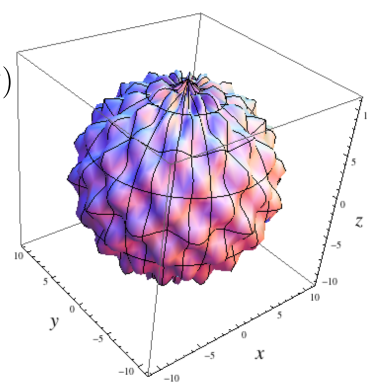


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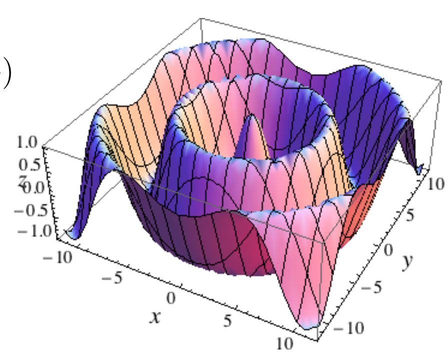
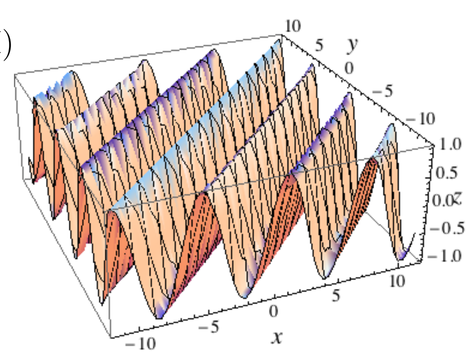



(H)


**2.** Find the following derivatives.

**a.** $f(x, y, z) = x^2 z^3 - xy + 2yz$. Find $f_x$ and $f_{zx}$.

**b.** $\frac{\partial}{\partial r} \cos(e^{rt} - r)$

**c.** Let $z$ be defined implicitly as a function of $x$ and $y$ by $y\, z\, e^{xz} = 5$.

Find a formula for $\frac{\partial z}{\partial x}$,

then evaluate $\frac{\partial z}{\partial x}$ when $x = 0$ and $y = 1$.

**3.** For the function

$$f(x, y) = \sqrt{5x^2 + y^2}\,,$$

find the linear approximation $L(x, y)$ (the linearization) of $f$ about the point $(1, 2)$.

**4.** For the function $g(x, y) = x^2 y$, and the point $P = (-2, 1)$,

**a.** Find a vector perpendicular to the level curve of $g$ through $P$.

**b.** Find the derivative of $g$ at $P$ in the direction of the vector $\langle 1, 3 \rangle$.

**c.** Find the derivative of $g$ at $P$ in the direction of the gradient $\nabla g$.

**d.** Find the derivative of $g$ at $P$ in the direction oriented at an angle of $2\pi/3$ to the gradient of $g$.

**5.** Find the derivative

$$\frac{d}{dt}L(x(t), y(t), t) \qquad \text{at} \quad t = 0 \tag{1}$$

given that

$$\begin{aligned} \nabla L(0,0,0) &= \langle 1,\ 2,\ -1 \rangle\,, \\ x(0) = 0\,, \quad \text{and} \quad y(0) &= 0\,, \\ x'(0) = 3\,, \quad \text{and} \quad y'(0) &= 5\,. \end{aligned} \tag{2}$$

**6.** Find a point on the level set (contour surface)

$$xy + yz - zx = 1$$

at which the tangent plane to the level set is parallel to the plane $y + 2z = 1$.

**7.** Find all *local minima*, *local maxima*, and *saddle points* of the function

$$f(x, y) = x^4 + y^4 - 4xy.$$

**8.** Find *all critical points* of the function

$$f(x, y, z) = xy + z$$

subject to the constraint

$$x^2 + 4y^2 + z^2 = 16$$

by using the method of Lagrange multipliers.

Then determine the *maximal and minimal values* of $f$ subject to the contraint.

(extra page)