Math 7320 @ LSU
Spring, 2018
Problem Set 1

Textbook problems refer to section number, subsection number, and problem number in V.I. Arnold's book "Ordinary Differential Equations".

1. Prove that

$$e^{A+B} = e^A\, e^B$$

if and only if $AB = BA$.

2. Verify the details of the computations for $e^{t\Lambda}$ at the bottom of page 6 of the lecture notes titled "General ideas of flows...".

3. Text: 14.8.2

4. Text: 14.9.1

5. Text: 14.9.2

6. Prove that the theorem in §14.9 is valid on larger functions spaces, such as the space of elementary functions, which is closed under differentiation. [Note Arnold's use of the symbol $\mathbb{R}^n$.] How broad can you make the space of functions on which $\exp(t\frac{d}{dx}) = H^t$ holds?

7. Prove that

$$\det e^A = e^{\mathrm{tr}A}$$

by using normal forms. Here, A is a linear operator in a finite-dimensional vector space over $\mathbb{R}$ or over $\mathbb{C}$.

8. Text: 18.4.1–3

9. Let A be an operator in a complex vector space of dimension n, and let ${}^{\mathbb{R}}A$ be the realification of A, acting on a real vector space of dimension $2n$. Prove that

$$\det e^{t\,{}^{\mathbb{R}}A} = \prod_{i=1}^{n} e^{2t\,\mathrm{Re}\lambda_i},$$

in which the λ_i are the eigenvalues of A (counted with multiplicity).