Math 7320 @ LSU
Spring, 2018
Problem Set 5

Textbook problems refer to section number, subsection number, and problem number in V.I. Arnold's book "Ordinary Differential Equations".

1. The Van der Pol oscillator is the ODE system

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x \;=\; 0\,.$$

Prove the following. For $\mu > 0$, $x \equiv 0$ is an unstable equilibrium solution and there exists a periodic solution; and for $\mu < 0$, all solutions converge to the stable equilibrium solution $x \equiv 0$.

2. Consider the discrete dynamical system in the interval $[0, 1]$ defined by iterating the "tent-map"

$$F(x) \;=\; 1 - |2x - 1|\,,$$

that is, with $x_0 \in [0, 1]$ given, a sequence $\{x_n\}_{n=0}^{\infty}$ in $[0, 1]$ is determined by

$$x_{n+1} = F(x_n)\,.$$

For each positive integer q, determine how many cycles (periodic orbits) of period q this system has. Prove that periodic orbits are dense in $[0, 1]$.