In answering 1-7 consider the following points. Can you calculate the requested partial using the formulas? If yes, can you shorten the calculation rather than proceeding naively? If you can’t use the formulas, then can you calculate using the definition?

Let \( f : \mathbb{R}^3 \rightarrow \mathbb{R} \) by
\[
f(x, y, z) = \begin{cases} 
\sin^3(x) + y \sin(z) & \text{if } (x, y, z) \neq (0, 0, 0) \\
0 & \text{if } (x, y, z) = (0, 0, 0)
\end{cases}
\]

1. Calculate \( D_1 f(0, 0, 0) \)
2. Calculate \( D_2 f(0, 0, 0) \)
3. Calculate \( D_3 f(0, 0, 0) \)
4. Calculate \( D_3 f(0, 1, 0) \)
5. Calculate \( D_2 f(\pi, \pi, \pi) \)
6. Calculate \( D_1 f(1, 0, 0) \)
7. If \( f \) is differentiable at \( (0, 0, 0) \), then what conclusion can you draw about \( f'(0, 0, 0) \)? Do you think \( f \) is differentiable at \( (0, 0, 0) \)? Why?

Let \( h : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) by \( h(x, y) = (xy, x + y^2) \).

8. Calculate all the \( D_i h_j \), the first order partial derivatives of \( h \).
9. Carefully prove that \( h \) is differentiable at every point \( (x, y) \in \mathbb{R}^2 \). You should not try to use the definition of the derivative.
10. What is \( h'(x, y) \)?

11. Let \( g : \mathbb{R}^{100} \rightarrow \mathbb{R} \) by
\[
g(x) = \sum_{i=1}^{99} x_i^2 x_{i+1}.
\]
Calculate \( D_{20} g(1, 2, 3, \cdots, 98, 99, 100) \).