Assignment I:
Spivak 1-2, 1-10, 1-12 and the following problem.
Class 1. Let \( x, y \in \mathbb{R}^n \) be nonzero. For the triangle with vertices 0, \( x, y \) (which is in the plane span\{\( x, y \)}), use the law of cosines to show that
\[
\langle x, y \rangle = |x||y|\cos \theta
\]
where \( \theta \) is the angle between \( x \) and \( y \).
Remarks:
1. If \( \langle x, y \rangle = 0 \) then we call \( x \) and \( y \) orthogonal or perpendicular.
2. The angle between \( x \) and \( y \) is \( \arccos(\langle x, y \rangle/|x||y|) \), cf., Spivak 1-8, 1-13.

Assignment II:

Assignment III:
Class 2. Suppose \( U \) is an open set in \( \mathbb{R}^n \), \( c \in U \), and \( f : U \to \mathbb{R}^m \). Show
\[
\lim_{x \to c} f(x) = 0 \iff \lim_{x \to c} |f(x)| = 0.
\]
Note that the zero in the first equation is \( 0 \in \mathbb{R}^n \) while the zero in the second equation is \( 0 \in \mathbb{R} \).
Spivak problems 2-1, 2-2, 2-7, and the following problem.
Class 3. Suppose \( f : \mathbb{R}^n \to \mathbb{R}^m \). Show that \( f \) is differentiable at \( a \in \mathbb{R}^n \) iff it satisfies the following property.
Property: There exists a linear map \( \lambda : \mathbb{R}^n \to \mathbb{R}^m \) and a function \( R : \mathbb{R}^n \to \mathbb{R}^m \) such that
\[
f(a + h) = f(a) + \lambda(h) + |h|R(h)
\]
where \( \lim_{h \to 0} R(h) = 0 \).
The property above is often given as the definition of the derivative of \( f \) at \( a \).
Assignment IV:
Spivak 2.10 acf, 2.11 ab, 2.17 abc, 2.18, 2.22, 2.24

Assignment V:
Spivak 2-17 dehi, 2-28, 2-29, 2-32. Part a was covered in Math 4031, please do it again! (You may wish to prove \( \lim_{x \to 0} x \sin(1/x) = 0 \).)

Assignment VI:
Updated with two additional questions and deadline extended
Class 4. Let \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) by \( f(x, y) = (x^3, y^3) \). Does \( f \) satisfy the hypotheses of the inverse function theorem on an open set containing (0,0)? If yes, then what conclusion can you draw. If not, then which hypotheses fail? Is there an open set containing (0,0) on which \( f \) is a one-to-one function?

Spivak 2-36 (In this problem, you are asked to prove that \( f \) is an open mapping. It is a significant consequence of the inverse function theorem), 2-38

Class 5. This problem may be viewed as 2-38 part c. Suppose \( f \) is the function from 2.38(b). Show that for all \( (x, y) \in \mathbb{R}^2 \) there is an open set \( O \) containing \( (x, y) \) on which \( f \) is 1-1.

Class 6. Discuss whether the curve whose equation is

\[
x^2 + y + \sin(xy) = 0
\]

can be described by an equation of the form \( y = f(x) \) on an open set containing the point (0,0). Also discuss if it be described by an equation of the form \( x = g(y) \) on an open set containing the point (0,0). Of course you should justify your answers.

Class 7. Let \( f : \mathbb{R} \to \mathbb{R} \) with \( f(1) = 0 \). Discuss the problem of solving the equation \( 2f(xy) = f(x) + f(y) \) for \( y \) near the point (1,1). Obtain the solution explicitly for \( f(t) = t^2 - 1 \). Do the same for the equation \( f(xy) = f(x) + f(y) \).

Assignment VII:
Spivak problems 3-2, 3-3, 3.9, 3-13

Assignment VIII:
Spivak problems 1-30, 3-14, 3-18, 3-22
Assignment IX:
Spivak problems 3-27 (You need to figure out the correct statement of the problem too), 3-28, 3-31 (You should, of course, prove your answer too.), 3-35