TOWARDS AN AUBRY - MATHER TYPE THEORY FOR PDE'S

I'll try to avoid any technicalities in at least the first two lectures and make them accessible to a general audience. Here's a brief description of the subject matter:

In the late 1980's, Moser initiated a study of a class of nonlinear elliptic PDE's in an attempt to develop a theory for them paralleling the work of Aubry and of Mather on dynamical systems. In these lectures we focus on a subclass of the equations Moser treated of the form:

$$-\Delta u + F_u(x,u) = 0, \quad x \in \mathbf{R}^n, \text{ where } F \in C^2(T^{n+1}, \mathbf{R}). \tag{1}$$

Related kinds of equations occur in Allen - Cahn models of phase transitions. We will describe a family of results obtained by Moser and others which show that possesses an enormous number of solutions which undergo single or multiple transitions and have certain minimality properties. The results are obtained using methods and ideas from the calculus of variations, PDE, dynamical systems, and geometry.