

## ABSTRACTS

All lectures will be held in the Business Education Complex, room 1510.

SATURDAY, APRIL 12

Morning

**Jean-Pierre Serre, Collège de France** *Modular forms mod 2*

Abstract: The modular forms of the title are of level 1, with coefficients in the field with 2 elements. They are polynomials in the series:

$$\Delta = q + q^9 + q^{25} + q^{49} + \dots$$

The corresponding Hecke operators have many interesting properties; I shall discuss some of them. This is joint work with J-L.Nicolas and J.Bellaïche; an outline of the main results can be found in three Comptes rendus notes published in 2012.

**Henri Darmon, McGill University** *The Birch and Swinnerton-Dyer conjecture for ring class characters of real quadratic fields*

Abstract: Let  $E$  be an elliptic curve over  $\mathbb{Q}$  and let  $\chi$  be a ring class character of a real quadratic field  $K$ . I will explain the proof that the non-vanishing of the central critical value  $L(E/K, \chi, 1)$  of the Hasse-Weil  $L$ -series of  $E$  twisted by  $\chi$  implies the triviality of the  $\chi$ -component of the Mordell-Weil group of  $E$ , in line with a natural Galois-equivariant refinement of the Birch and Swinnerton-Dyer conjecture. The proof relies on Gross-Kudla-Schoen diagonal cycles and on their variation in  $p$ -adic families. Possible applications to the theory of “Stark-Heegner points” and to explicit class field theory for real quadratic fields will also be evoked. All of this is joint work with Victor Rotger.

**Ken Ribet, University of California at Berkeley** *Eisenstein primes at composite levels*

Abstract: The talk will describe work by the speaker and by Hwajong Yoo on “Eisenstein issues” for Jacobians of modular curves. In the case of prime level (i.e., for the abelian variety  $J_0(p)$ , where  $p$  is a prime), Mazur’s landmark “Eisenstein ideal” article provided definitive answers to essentially all possible questions. Even when  $p$  is replaced by a product of two distinct primes, apparently simple questions are not guaranteed to have a simple answer. Some questions are unresolved.

SATURDAY, APRIL 12

Afternoon

**Dorian Goldfeld, Columbia University** *First moments of Fourier coefficients of  $GL(n)$  cusp forms*

Abstract: Let  $\pi$  be an irreducible cuspidal automorphic representation of  $GL(n, \mathbb{A})$  where  $n > 1$  and  $\mathbb{A}$  is the adèle ring of  $\mathbb{Q}$ . Let  $a_\pi(n)$  denote the  $n^{\text{th}}$  Dirichlet series coefficient of the  $L$ -function associated to  $\pi$ . The main goal is to obtain strong bounds for the first moment

$$\sum_{n \leq x} a_\pi(n).$$

The bounds we obtain are better than all previously obtained bounds for the higher rank situation when  $n > 2$ . This is joint work with Jyoti Sengupta.

**Kristin Lauter, Microsoft Research** *On singular moduli for arbitrary discriminants*

Abstract: We will present a generalization of Gross and Zagier's theorem on the factorization of differences of singular moduli to the case of arbitrary discriminants. This is joint work with Bianca Viray.

**Peter Sarnak, Princeton University** *Families of  $L$ -functions and their symmetry*

Abstract: We give a working definition of a family of  $L$ -functions of automorphic forms on  $GL(n)/\mathbb{Q}$ . Associated with the family is a "Sato-Tate" measure and with it certain indicators which dictate the distribution of the zeros of the  $L$ -functions in the family. We show that the last fall into exactly four universality classes (as was found by Katz and Sarnak in the 90's in the case of function fields). This is joint work with S.Shin and N.Templier.

SATURDAY, APRIL 12

Evening

The banquet will be in the Atchafalaya Room (339 LSU Union, 3rd floor), Southwest corner, in back of the building. The banquet will be from 5:30-7:30, with a cash bar (wine and beer only) from 5:30 -6:30.

SUNDAY, APRIL 13

Morning

**Kathrin Bringmann, University of Cologne, Germany** *Negative index Jacobi forms and quantum modular forms*

Abstract: We consider the Fourier coefficients of a special class of meromorphic Jacobi forms of negative index. Much recent work has been done on such coefficients in the case of Jacobi forms of positive index, but almost nothing is known for Jacobi forms of negative index. In recent joint work with Thomas Creutzig and Larry Rolen I show that their Fourier coefficients have a simple decomposition in terms of partial theta functions. In particular, we find a new infinite family of rank-crank type PDEs generalizing the famous example of Atkin and Garvan. We then describe the modularity properties of these coefficients, showing that they are “mixed partial theta functions”, along the way determining a new class of quantum modular partial theta functions which is of independent interest.

**Benedict Gross, Harvard University** *Newforms*

Abstract: The classical theory of newforms gives distinguished vectors in cuspidal representations of  $\mathrm{PGL}(2)$ . We generalize this theory to generic cuspidal representations of odd orthogonal groups. The distinguished vectors are fixed by certain compact subgroups of the adelic orthogonal group.

**Ken Ono, Emory University** *A framework of Rogers-Ramanujan identities*

Abstract: In his first letter to G. H. Hardy, Ramanujan offered surprising evaluations of a  $q$ -continued fraction. Hardy later recalled his reaction to these formulas:

“These formulas defeated me completely...they could only be written down by a mathematician of the highest class. They must be true because no one would have the imagination to invent them”. –G. H. Hardy

Ramanujan had a secret device, the two Rogers-Ramanujan identities, which are now ubiquitous in mathematics. It turns out that these identities and Ramanujan’s theory of evaluations are hints of a much larger theory. In joint work with Michael Griffin and Ole Warnaar, the author has discovered a rich framework of Rogers-Ramanujan identities, one which comes equipped with a beautiful theory of algebraic numbers. The story blends the theory of Hall-Littlewood polynomials, modular forms, and the representation theory of Kac-Moody affine Lie algebras.

MONDAY, APRIL 14

Morning

**John Tate, Harvard University and University of Texas at Austin** *On Dwork's function  $C$*

Abstract: Suppose  $(E, \omega)$  is an elliptic curve with differential, defined over a  $p$ -adic field  $k$ , with ordinary reduction. In 1957 I challenged B. Dwork to prove the existence of a certain constant  $C$  attached to  $(E, \omega)$  which lives in the completion of the maximal unramified extension of  $k$ . Dwork realized that the constant  $C$  should be viewed as a function, and studied it for the Legendre family  $(E_\lambda, \omega_\lambda)$ , finding that for that family, the function  $C = C(\lambda)$  is hypergeometric. I will discuss less explicit analogs of Dwork's results for arbitrary families.

**Yifan Yang, National Chiao Tung University, Taiwan** *Special values of hypergeometric functions and periods of CM elliptic curves*

Abstract: Let  $X$  be the Shimura curve associated to a maximal order in the indefinite quaternion algebra of discriminant 6 over  $\mathbb{Q}$ . By realizing modular forms on  $X$  in two ways, in terms of hypergeometric functions and in terms of Borcherds forms, respectively, and using Schofer's formula for values of Borcherds forms at CM-points, we obtain special values of certain hypergeometric functions in terms of periods of CM elliptic curves.

**Ching-Li Chai, University of Pennsylvania** *A scheme-theoretic definition of leaves and Serre-Tate coordinates*

Abstract: The original definition of leaves, introduced by Oort in 1999, based on the idea of geometric constancy for an equi-characteristic  $p$  family of  $p$ -divisible groups. It is difficult to study differential properties of leaves with such a "pointwise" definition. In this talk we will explain a scheme-theoretic definition of "geometric constancy" and "leaves", obtained in collaboration with F. Oort. As an application, we will explain a "group-like" structure on local coordinates of leaves, generalizing a classical theorem of Serre and Tate.

**Gabriele Nebe, RWTH Aachen University, Germany** *Hecke operators for lattices and codes*

Abstract: In 1957 Martin Kneser described a procedure to enumerate all lattices in a given genus and applied it to the classification of unimodular lattices in small dimension. The idea is to define a neighboring relation on the lattices in a genus and call  $L$  adjacent to  $M$  if their intersection has index  $p$  for a fixed prime  $p$ . This yields a linear operator on the space of formal linear combinations of isometry classes of lattices in the genus, for which Yoshida (in 1984) has computed the explicit expression as an element in the Hecke algebra acting on the space of Siegel theta series. I will present a coding theory analogue of this operator.

MONDAY, APRIL 14

Afternoon

**Shou-Wu Zhang, Princeton University** *p-adic Waldspurger Formula*

Abstract: In this talk, I will explain a  $p$ -adic Waldspurger formula proved by Bertolini–Darmon–Prasanna under Heegner condition, and in full generality later by Liu–Zhang–Zhang. I will start with a classical Waldspurger formula on complex modular forms and a Gross–Zagier formula on rational modular forms, then define  $p$ -adic modular forms,  $p$ -adic L-functions,  $p$ -adic period integrals, and finally state a  $p$ -adic Waldspurger formula.

**Jennifer Balakrishnan, University of Oxford, United Kingdom** *p-adic heights and integral points on curves*

Abstract: We discuss explicit computations of  $p$ -adic line integrals (Coleman integrals) on elliptic and hyperelliptic curves and some applications. In particular, we relate a formula for the component at  $p$  of the  $p$ -adic height pairing to a sum of iterated Coleman integrals. We use this to give a Chabauty-like method for computing  $p$ -adic approximations to integral points on such curves when the Mordell-Weil rank of the Jacobian equals the genus. This is joint work with Amnon Besser and Steffen Müller.

**Tong Liu, Purdue University** *Noncongruence Modular forms and automorphy of Scholl representations*

Abstract: Scholl constructed Galois representations attached to noncongruence modular forms (i.e., modular forms of noncongruence subgroups inside  $SL_2(\mathbb{Z})$ ). These representations play a fundamental role in the arithmetic of noncongruence modular forms. Langlands' philosophy predicts that Scholl representations should be automorphic (arise from certain automorphic forms). Recently, the automorphy of Scholl representations has shown to shed new lights on noncongruence modular forms. In this talk, we will explain how to use modern technique of automorphy lifting theorems to establish the automorphy of Scholl representations. In particular, we provide several situations that Scholl's representations can be proved to be (potentially) automorphic. This is joint work with Wen-Ching Winnie Li and Ling Long.

**Mingsuan Kang, National Chiao Tung University, Taiwan** *L-functions and Geometric Zeta Functions of uniform lattices*

Abstract: The classical Ihara zeta function on a torsion-free uniform lattice  $\Gamma$  of  $G = \mathrm{PGL}_2(\mathbb{Q}_p)$  can be regarded as a geometric zeta function on the building of  $G$  quotient by  $\Gamma$ . It is related to a local  $L$ -function attached to the representation  $L^2(\Gamma \backslash G)$  via the trace formula and Satake isomorphism. One can try to generalize Ihara zeta functions to torsion-free uniform lattices in general  $p$ -adic reductive Lie groups, but only very few results are known up to date. On the other hand, from the philosophy of the field with one element, the general theory shall also hold for the case  $p = 1$ . In this case, the  $p$ -adic reductive Lie group is replaced by its affine Weyl group and the building becomes a single apartment. In this talk, we will discuss geometric zeta functions on the finite quotients of the building of an affine Weyl group and their relations to  $L$ -functions arising from representations.

TUESDAY, APRIL 15

Morning

**Ram Murty, Queen's University** *Automorphy and the Sato-Tate conjecture*

Abstract: We show how a special case of the Langlands functoriality conjecture leads to the automorphy of symmetric power  $L$ -functions attached to a cuspidal automorphic representation of  $GL_2(\mathbb{A}_{\mathbb{Q}})$ . This is joint work with Kumar Murty.

**Chian-Jen Wang, Tamkang University, Taiwan** *Zeta functions of complexes from  $Sp(4)$*

Abstract: Let  $F$  be a non-archimedean local field. In recent joint work with Yang Fang and Wen-Ching Winnie Li, we introduced certain zeta functions associated to a finite complex  $X_{\Gamma}$  arising as the quotient of the Bruhat-Tits building  $X$  associated to  $Sp_4(F)$  by a discrete torsion-free cocompact subgroup  $\Gamma$  of  $PGSp_4(F)$ , and gave closed form expressions for these zeta functions. In this talk, we shall extend these results to include another family of zeta functions associated to  $X_{\Gamma}$  and examine their connections to local  $L$ -functions for  $GSp_4(F)$ .

**YoungJu Choie, POSTECH, Korea** *An explicit bound for the first sign change of the Fourier coefficients*

Abstract: Recently there are many results concerning sign changes of the Fourier coefficients of elliptic cusp forms. In this talk we discuss an explicit upper bound for the first sign change of the Fourier coefficients of an arbitrary non-zero Siegel cusp form  $F$ .

**Robert Osburn, University College Dublin, Ireland** *Supercongruences*

Abstract: It is known that the numbers which occur in Apéry's proof of the irrationality of  $\zeta(2)$  and  $\zeta(3)$  have many interesting arithmetic properties. In particular, work of Beukers and Coster show that they satisfy two-term and modular supercongruences. In this talk, we discuss recent progress and future directions concerning this and other types of supercongruences. This is joint work with Brundaban Sahu (NISER) and Armin Straub (UIUC).

TUESDAY, APRIL 15

Afternoon

**Jeffrey Lagarias, University of Michigan** *The Lerch Zeta Function and Generalizations*

Abstract: The Lerch zeta function is a three-variable generalization of the Riemann zeta function. This talk first describes work done with Wen-Ching Winnie Li on algebraic and analytic structures associated to this function. These structures include a multivalued analytic continuation of this function in three complex variables, and its relation to functional equations and differential equations satisfied by this function. We describe two-variable Hecke operators for which the Lerch zeta function is a simultaneous eigenfunction, and characterize it by these properties. We conclude with recent work giving an automorphic interpretation of this function and generalizations.

**George E. Andrews, The Pennsylvania State University** *Congruences for Fishburn Numbers*

Abstract: This talk is on joint work with James Sellers. The Fishburn numbers,  $\xi(n)$ , are defined to be the number of upper triangular matrices with nonnegative integer entries, without zero row or column sums such that the sum of the entries is  $n$ . The Fishburn numbers grow superexponentially so that their generating function only converges at zero. We prove that if  $p$  is prime and a quadratic nonresidue modulo 23, then there is a nonempty set,  $T(p)$ , of integers in  $[0, p - 1]$  such that if  $j$  is in  $T(p)$ , then  $\xi(pn + j)$  is congruent to 0 modulo  $p$  for all nonnegative integers  $n$ .