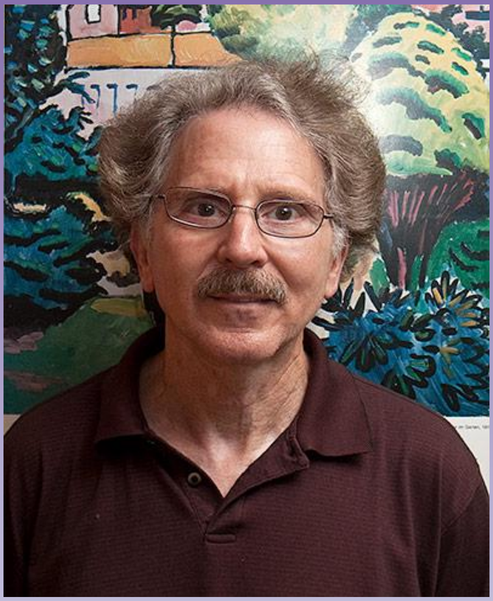
Southern Regional Number Theory Conference

in Honor of Robert Perlis on his Retirement

April 8-9, 2017
Louisiana State University
Baton Rouge, Louisiana



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Noriko Yui Queen's University (Canada)

Robert Perlis received his Ph.D. from MIT in 1972 and has served on the LSU faculty since 1980. While at MIT, he was known as “the graduate student with a dog”, Bartok, “and a kid”, Alexander, both of whom accompanied him to school every day. At LSU, he directed the dissertations of seventeen students, including eight women and three African-Americans. He served as a program officer in the division of mathematical sciences at NSF from 1999 to 2000, and as departmental chair from 2010 to 2016.

Organizers: Luca Candelori, Jerome Hoffman, Ling Long, Karl Mahlburg, Fang-Ting Tu

The Number Theory Foundation

National Science Foundation

LSU Math Depart. & College of Science

To register: https://www.math.lsu.edu/~lcandelori/2017SouthernRegional.html

Biography of Robert Perlis

I was born in 1946 shortly after my family moved to Purdue University where my father was a math professor. I attended Purdue University as an undergraduate before going to MIT for graduate school. This was during the war in Vietnam, and I turned down an NSF graduate fellowship in order to accept an MIT teaching assistantship which brought a draft deferment. While at MIT, I was known as ``the graduate student with a dog", Bartok, ``and a kid", Alexander, both of whom accompanied me to school every day.

In my first year I attended the lectures of N. C. Ankeny on algebraic number theory, and fell in love with the subject. Jurgen Neukirch spent the following year as a visitor at MIT and gave a second-year course on algebraic number theory from the point of view of valuations. He invited me to return with him to Regensburg, Germany, but I couldn't accept at the time due to the draft. But he left me with a problem that I solved for my doctoral dissertation, formally under Ankeny. By 1972 the draft was by lottery and my number was high enough not to be called, so I was able to go to Regensburg as a post-doctoral assistant to Neukirch. At Neukirch's suggestion, I began to think about pairs K, L of number fields with identical Dedekind zeta functions. I was able to show that K and L shared many properties in common, and I called K and L ``arithmetically equivalent". That name became very popular, and I received many invitations to talk on the subject. After many years of work, Bart de Smit and I were able to give an example showing that arithmetically equivalent fields did not have to have the same class numbers. In 1975 I went to Bonn University for a one-year postdoc at the SFB 40, a precursor to the present day Max Planck Institute. After one year, the director Fritz Hirzebruch invited me to be his assistant, but the German Labor Department would not approve it. For the next four years, without requesting it, I received a one-year extension from the SFB. I believe I was the longest short-term visitor in SFB history. In 1980 I accepted an offer to come to LSU as an associate professor. Here I met Pierre Conner, who asked me if I knew any interesting quadratic forms associated with algebraic number fields. This began a collaboration that bore lots of fruit, including our book on trace forms. The trace form of a field L over a subfield K is a quadratic form that naturally lives in a ring called the Witt ring of K. Later we became interested in the Witt rings themselves. Together with Rick Litherland and Kaz Szymiczek, and based on work of Jenna Carpenter, we proved a local-global principle for Witt rings of algebraic number fields.

In 1985, T. Sunada showed that a construction analogous to the one I used to produce pairs of arithmetically equivalent number fields could be used to produce pairs of non-isometric isospectral Riemannian manifolds. As a special case, this construction produces pairs of graphs with identical Ihara zeta functions. Along with some REU students, I wrote a few papers related to this latter subject. At LSU, I directed the dissertations of seventeen students, including eight women and three African-Americans. I served as a program officer in the division of mathematical sciences at NSF from 1999 to 2000, and as departmental chair from 2010 to 2016. I enjoyed every moment of my mathematical life so far, and I look forward to continued involvement during my retirement.

TITLES AND ABSTRACTS

Zeta functions of graphs and Kirchhoffian indices
Marius Somodi, University of Northern Iowa

The distance between two vertices of a finite graph is typically defined as the length (number of edges) of the shortest path between the two vertices. About 25 years ago, a new distance function, called the resistance distance, was introduced by regarding the graph as an electrical network with a unit resistor on each edge. Subsequently, three resistance distance based graph invariants were defined: the Kirchhoff index, the multiplicative degree-Kirchhoff index, and the additive degree-Kirchhoff index.

The Ihara zeta function of a graph is a function of complex argument defined using the prime cycles of the graph. For connected graphs without vertices of degree 1, the zeta function is known to encode several invariants of the graph, including its numbers of vertices, edges, and loops. In this talk, we take a closer look at Kirchhoffian indices of graphs that have the same Ihara zeta function.

Mirror symmetry for elliptic curves
Noriko Yui, Queen's University, Canada

Mirror symmetry is a theory developed for pairs of Calabi-Yau threefolds. In this talk, we will consider a toy model of mirror symmetry phenomena for elliptic curves.

We look into the formula, due to Douglas and Dijkgraaf, on the generating function, $F_g(q)$, of the number of simply ramified covers of genus $g \geq 1$ over a fixed elliptic curve with marked points. Their result is that $F_g(q)$ is a quasimodular form of weight $6g-6$ on the full modular group $PSL_2(\mathbf{Z})$.

There are two ways of computing $F_g(q)$: the fermionic count and the bosonic count. The fermionic counting is a mathematical theory and we can give a rigorous mathematical proof. However, the bosonic counting rests on physical arguments, which involves path integrals on trivalent Feynman diagrams. We will compute $F_g(q)$ with the bosonic approach for small genera.

This establishes the mirror symmetry for elliptic curves.

Recent Developments in Gassmann Equivalence
Bir Kafle, Purdue University Northwest

In 1926, ETH Zurich student Fritz Gassmann published a paper explaining some unpublished work of Adolf Hurwitz. He reformulated Hurwitz's initial attempts to the following condition. Let G be a group and let H, H' be subgroups of G such that each conjugacy class of G intersects H and H' in the same number of elements. Today, this condition is called Gassmanns condition and subgroups satisfying this condition are called Gassmann equivalent in G. In this talk, we present some recent developments we have made in Gassmann equivalence and their application in other areas of mathematics.

The method of brackets

Victor Moll, Tulane University

This is a formal procedure to evaluate definite integrals coming from some Feynman diagrams. It is a based upon a small number of rules, some of which have been established. The talk will present a collection of examples to illustrate the flexibility of the method.

Indecomposable vector bundles over genus zero modular curves and vector-valued modular forms

Luca Candelori, Louisiana State University

For each genus zero Fuchsian group, its corresponding modular curve is an orbifold curve with finitely many orbifold points of cyclic order. The existence of indecomposable vector bundles over such curves was established by Geigle-Lenzing and Crawley-Boevey, who have shown that they are indexed by the roots of a Kac-Moody Lie algebra G constructed from the group. In this talk we show how to explicitly construct some of these indecomposable vector bundles using modular forms. As an application, we study the structure of graded modules of vector-valued modular forms over genus zero fuchsian groups: in particular, we prove that these modules decompose as direct sums of (indecomposable) maximal Cohen-Macaulay modules, indexed by the roots of the Kac-Moody Lie algebra G.

Solitary Riemann surfaces

Peter Buser, École Polytechnique Fédérale de Lausanne, Switzerland

An algebraic number field K is called solitary if no other algebraic number field has the zeta function K has (Perlis, 1977). There is a similar concept for Riemann surfaces, and as in the case of number fields one aims at describing classes of examples that are solitary respectively aren't. After a brief history a class of examples will be described whose solitariness is encoded in the zeta function by some kind of Gödel numbering.

Congruent numbers and ternary quadratic forms

Hourong Qin, Nanjing University, China

A positive integer is called a congruent number if it is the area of a right-angled triangle, all of whose sides have rational length. A celebrated theorem due to Tunnell gives a criterion for a positive integer to be congruent (under the BSD). We show that if a square-free and odd (respectively, even) positive integer n is a congruent number, then

$$\#\{(x,y,z)\in\mathbb{Z}^3 | n = x^2+2y^2+32z^2\} = \#\{(x,y,z)\in\mathbb{Z}^3 | n = 2x^2+4y^2+9z^2-4yz\},$$

respectively,

$$\#\{(x,y,z)\in\mathbb{Z}^3 | \frac{n}{2} = x^2+4y^2+32z^2\} = \#\{(x,y,z)\in\mathbb{Z}^3 | \frac{n}{2} = 4x^2+4y^2+9z^2-4yz\}.$$

If we assume that the weak Brich-Swinnerton-Dyer conjecture is true for the elliptic curves $E_n : y^2 = x^3 - n^2x$, then, conversely, these equalities imply that n is a congruent number.

We shall also discuss some applications.

On the homology of SL_n

Marco Schlichting, University of Warwick , UK

Matsumoto and Moore gave a presentation of $H_2(SL_2F)$ for any (infinite) field which is rather complicated. The analogous presentation of $H_2(SL_nF), n > 2$, is very simple and lead to the definition of Milnor K-theory and its relation to quadratic forms and etale cohomology culminating in Voevodsky's proof of a conjecture of Milnor. In this talk I will give a simple presentation of $H_2(SL_2R)$ for any local ring R (with infinite residue field) which leads to the computation of $H_n(SL_nR, SL_{n-1}R)$ as the n-th Milnor Witt K-group of R. If time permits I will explain an application to Euler classes of projective modules as obstruction for splitting off a rank 1 free direct summand. The talk is based on arXiv:1502.05424 [math.KT].