Erdős-Kac Theorem in Short Intervals

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Let $\omega(n)$ denote the number of distinct prime divisors of $n$. The celebrated Erdős-Kac theorem asserts that

$$\lim_{x\to\infty}\frac{1}{x}\#\left\{x\le n\le 2x : \frac{\omega(n)-\log\log n}{\sqrt{\log\log n}}\le t\right\}=\int_{-\infty}^{t} e^{-\frac{y^2}{2}}dy$$

for any $t\in\mathbb{R}$. Recognizing that the right-hand side is the Gaussian distribution, one may regard the Erdős-Kac theorem as a "Central Limit Theorem." In this context, one can further view $\omega(n)$ as a "random variable" defined on the probability space $[x, 2x]$. From this point of view, one may be curious about what will happen if the probability space becomes "smaller." More precisely, what can one say about the limiting behavior of

$$\frac{1}{h}\#\left\{x\le n\le x+h : \frac{\omega(n)-\log\log n}{\sqrt{\log\log n}}\le t\right\}? \qquad (1)$$

In this talk, we will show that the limiting behavior of (1) is still Gaussian for $h=x^\theta$ with $0<\theta\le 1$. If time permits, we will discuss a prime analogue of (1) in light of the work of Halberstam.

This is joint work with Dr. Peng-Jie Wong.