New Results on Polynomials in Difference Sets

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It is a well known result in arithmetic combinatorics, established independently by Sárközy and Furstenberg, that a set of integers with positive upper density must contain two distinct elements that differ by a perfect square. The best-known quantitative bounds for this result were established with an intricate Fourier analytic argument by Pintz, Steiger, and Szemerédi. In this talk, we discuss the extension of these bounds from perfect squares to, at long last, the largest possible class of single-variable polynomials, as well as even better bounds for a large class of two-variable polynomials. In both settings, we utilize a polynomial-specific sieve as a bridge to optimal exponential sum estimates (due to Weil and Deligne) over finite fields. This includes joint work with John Doyle.