The Valuation of Polynomial Sequences

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Given a prime p and any positive integer n, the p-adic valuation of n, denoted by $\nu_p(n)$, is the highest power of p that divides n. This notion is extended to $\mathbb{Q}$ by $\nu_p\left(\frac{a}{b}\right) = \nu_p(a) - \nu_p(b)$ and by setting $\nu_p(0) = \infty$.

For any sequence $\{a_n\}$ and a fixed prime p, the sequence of valuations $\{\nu_p(a_n)\}$ often presents interesting challenges. One of the goal is to obtain a closed form for these valuations.

In this talk I will discuss p-adic valuations of polynomials, sequences $\{P(n)\}$, focusing on polynomials of low degree $(2, 3)$. It will be shown that the sequence of valuation $\nu_2(P(n))$ can be represented as a binary tree. Sometimes this is a finite tree. Examples of trees with infinite branches will be presented. The number of these branches is shown to be connect to the number of roots of $x^n - l$ in the ring of p-adic integers $\mathbb{Z}_p$. This is a joint work with Diego Villamizar.