Scarcity of Congruences for Colored Partition Functions

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The arithmetic properties of the ordinary partition function $p(n)$ have been a topic of intensive study ever since Ramanujan proved that for all integers n, $p(Qn+t) \equiv 0 \pmod{Q}$ for the primes $Q = 5, 7, 11$, where $24t \equiv 1 \pmod{Q}$. Today it is known that there are many congruences of the form $p(Qmn+t) \equiv 0 \pmod{Q}$ for all n, where Q is prime and $m \geq 5$. In joint work with Scott Ahlgren and Martin Raum, we prove that such congruences where m is prime are scarce in a certain sense, if any exist at all, unless a certain nonzero cusp form has a large number of coefficients which are divisible by Q.

On the other hand, ongoing joint work with Alex Caione, Jack Chen, Maddie Diluia, Oscar Gonzalez, and Jamie Su shows that there are infinitely many congruences of this type for the colored partition functions, and all of them appear to arise from a congruence modulo Q between a generating function for certain colored partition numbers and a theta function.