Sums of Powers of Binomials, Their Apéry Limits, and Franel's Suspicions

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Apéry's proof of the irrationality of $\zeta(3)$ relies on representing that value as the limit of the quotient of two rational solutions to a three-term recurrence. We review such Apéry limits and explore a particularly simple instance. We then explicitly determine the Apéry limits attached to sums of powers of binomial coefficients. As an application, we prove a weak version of Franel's conjecture on the order of the recurrences for these sequences. This is based on joint work with Wadim Zudilin.