

BOUNDS ON THE TORSION SUBGROUPS OF NÉRON-SEVERI GROUPS

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ABSTRACT. Let $X \hookrightarrow \mathbb{P}^r$ be a smooth projective variety defined by homogeneous polynomials of degree $\leq d$ over an algebraically closed field k . Let $\mathbf{Pic} X$ be the Picard scheme of X , and $\mathbf{Pic}^0 X$ be the identity component of $\mathbf{Pic} X$. The Néron–Severi group scheme of X is defined by $\mathbf{NS}X = (\mathbf{Pic} X)/(\mathbf{Pic}^0 X)_{\text{red}}$, and the Néron–Severi group of X is defined by $\text{NS}X = (\mathbf{NS}X)(k)$. We give an explicit upper bound on the order of the finite group $(\text{NS}X)_{\text{tor}}$ and the finite group scheme $(\mathbf{NS}X)_{\text{tor}}$ in terms of d and r . As a corollary, we give an upper bound on the order of the torsion subgroup of second cohomology groups of X and the finite group $\pi_{\text{ét}}^1(X, x_0)_{\text{tor}}^{\text{ab}}$. We also show that $(\text{NS}X)_{\text{tor}}$ is generated by $(\deg X - 1)(\deg X - 2)$ elements in various situations.