Runs of Integers with the Same Number of Divisors

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Abstract

In 1952, Erdős and Mirsky asked for an upper bound for the longest sequence of consecutive integers $\leq x$ with the same number of divisors. We find a new upper bound for this quantity, building off recent work of Spătaru. We also consider the related question of finding the largest k such that $d(n+1) \geq d(n+2) \geq \cdots \geq d(n+k)$ for some $n \leq x-k$. Finally, we consider variants of this problem for different arithmetic functions such as the sum-of-proper-divisors function s(n) and the Carmichael λ function. In particular, we show that $s(n+1) = s(n+2) = \cdots = s(n+6)$ has no solutions. This is joint work with Joseph Vandehey.