

CONGRUENCES FOR SOME PARTITION FUNCTIONS AND THEIR ℓ -ADIC PROPERTIES

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ABSTRACT. In 2003, Ahlgren and Boylan proved that Ramanujan congruences only occur at primes 5, 7, 11. In 2012, Folsom-Kent-Ono conceptually proved this by calculating a bound for the rank of a ℓ -adic module related to the generating function of the partition function. Later, Boylan-Webb gave more comprehensive proof of this work.

In this presentation, I will talk about the congruences and ℓ -adic properties of a two-parameter partition function $p_{[1^c \ell^d]}(n)$. We define this partition function, for any integers c, d by

$$\sum_{n=0}^{\infty} p_{[1^c \ell^d]}(n) q^n = \prod_{n=1}^{\infty} \frac{1}{(1 - q^n)^c (1 - q^{\ell n})^d}.$$

From this work, we were able to determine why Ramanujan congruences only occur at certain primes for a large class of partition functions, including ℓ -regular partitions, and ℓ -core partitions.