# GENERALIZED ARITHMETIC PROGRESSIONS AND DIOPHANTINE APPROXIMATION BY POLYNOMIALS

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Abstract. We discuss two related notions of "approximate subgroups" inside finite sets of integers: Bohr sets, which capture simultaneous Diophantine approximation, and symmetric generalized arithmetic progressions (GAPs). For example, fix natural numbers $N$ and $d$ and consider the following pair of questions:

(1) For fixed $\alpha_1,\dots,\alpha_d \in \mathbb{R}$, how close to an integer can we simultaneously make $n^2\alpha_1,\dots,n^2\alpha_d$ for some $1 \leq n \leq N$?
(2) How large can a set of the form $\{x_1\ell_1+\cdots+x_d\ell_d : -L_i \leq \ell_i \leq L_i\} \subseteq [-N,N]$ be before it is guaranteed to contain a perfect square?

Our discussions range from classical facts like the Kronecker approximation theorem and Linnik's theorem, to a recent breakthrough result of Maynard and its potential future applications. In between we survey results including previous joint work with Neil Lyall and Ernie Croot.