THE EULER TOTIENT FUNCTON ON LUCAS SEQUENCES

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ABSTRACT. In 2009, Luca and Nicolae proved that the only Fibonacci numbers whose Euler totient function is another Fibonacci number are 1, 2, and 3. In 2015, Faye and Luca proved that the only Pell numbers whose Euler totient function is another Pell number are 1 and 2. Here we add to these two results and prove that for any fixed natural number $P \ge 3$, if we define the sequence $(u_n)_n$ as $u_0 = 0$, $u_1 = 1$, and $u_n = Pu_{n-1} + u_{n-2}$ for all $n \ge 2$, then the only solution to the Diophantine equation $\varphi(u_n) = u_m$ is $\varphi(u_1) = \varphi(1) = 1 = u_1$.