

APPLICATIONS OF BELYI MAPS IN ARITHMETIC DYNAMICS

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ABSTRACT. Unicritical polynomials, typically written in the form $z^d + c$, have been widely studied in arithmetic and complex dynamics and are characterized by their one finite critical point. The behavior of a map's critical points under iteration often determines the dynamics of the entire map. Rational maps where the critical points have a finite forward orbit under iteration are called post-critically finite (PCF), and these are of great interest in arithmetic dynamics. The family of (single-cycle normalized) dynamical Belyi polynomials have two fixed critical points, so they are PCF by construction. These maps provide a new testing ground for conjectures and ideas related to post-critically finite polynomials. Using this family, we can begin to explore properties of polynomials with two critical points. In this talk we will discuss how the family of dynamical Belyi polynomials connects to the more general setting of bicritical polynomials and how we can use it to classify PCF polynomials, to answer a question of Silverman pertaining to the height of critically fixed maps, and to determine reduction properties of PCF maps. In particular, we will demonstrate that considering maps with only one additional critical point may be enough to provide complete answers to questions in arithmetic dynamics.