Name:  

Homework Set One: Math 7510

1. Let $K$ be a one-dimensional simplicial complex satisfying the one-manifold condition: Every zero simplex is contained in exactly two one-simplices. Compute $\alpha_0 - \alpha_1$. (Hint: You made need to add/define an additional condition to formulate a concise answer.)

2. For a given simplicial complex $K$, show that the number of $k$-simplices satisfies $\alpha_k \leq \binom{|V|}{k+1}$. (See notes for further details.)

3. Let $K$ be a two-dimensional simplicial complex whose vertex set can be inscribed in a cylinder, $Cyl = \{(x, y, z) : x^2 + y^2 = 1, -1 \leq z \leq 1\}$ such that the projection of the geometric simplices, $\pi : |K| \rightarrow Cyl$ is a bijection. Compute the Euler characteristic of $K$.

4. Find a triangulation of the torus: $\{(x, y, z) : distance((x, y, z), (x, y, 0)/\sqrt{x^2 + y^2}) = 1/2\}$ and compute its Euler Characteristic. (Bonus: Find a triangulation with exactly 7 vertices and 14 triangles and prove this is minimal.)