1. Definitions

Definition 1.1 (i). An (abstract) finite simplicial complex is a pair, 
\[ K = (V, C \subset 2^V := \{ S : S \subset 2^V \} = \{0, 1\}^V = \{ \varphi : V \to \{0, 1\} \}) \]
such that

i) \( \{v\} \subset C \) for all \( v \in V \).

ii) If \( S_1 \subset S_2 \in C \), then \( S_1 \in C \).

The relationship between functions from \( V \) to \( \{0, 1\} \) and subsets of \( V \) is the correspondence given by \( S = \varphi^{-1}(1) \).

Definition 1.2 (ii). A finite simplicial complex is a pair, \( K = (V, C \subset 2^V) \) where \( V \) is a finite set of elements called vertices and \( C \) is a set of simplices (subsets of \( V \)) satisfying:

i.) \( \{v\} \in C \) for all \( v \in V \) and,

ii.) if \( S_1 \in S_2 \in V \) then \( S_1 \in C \)

Definition 1.3. The simplicial closure of \( D \) is

\[ D = \{ S : S \subset D \in D \} \cup \{ \{v\} : v \in V \} \].

Definition 1.4. An element of \( C \), (i.e. a subset of \( V \) with \((i + 1)\) elements) will be called an \( i \)-simplex. The number of \( i \)-simplices in a simplicial complex will be denoted \( \alpha_i \) and the set of \( i \)-simplices by \( V_i \).

Note: \( V_1 = V \), the set of vertices.

Definition 1.5. The standard \( n \)-simplex is

\[ \Delta_n = (V = [n] = \{0, 1, 2, \ldots, n\}, C = 2^{[n]}) \].

Definition 1.6. The convex hull of a set of points \( S \) in \( n \)-dimensions is the intersection of all convex sets containing \( S \).

Definition 1.7. Geometrically, the standard \( n \)-simplex is often identified with the convex hull of the standard ordered basis of \( \mathbb{R}^{n+1} \).

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Definition 1.8. The geometric $n$-simplex is:

$$|\Delta_n| := \left\{ \bar{x} \in \mathbb{R}^{n+1} : \bar{x} = (t_1, \ldots, t_{n+1}), \sum_{i=0}^{n} t_i = 1, 1 \geq t_i \geq 0 \right\}$$

1.1. Theory. Two Variants of Simplicial Complexes:

i) Abstract

ii) Geometric

2. Examples

([3], \{0\}, \ldots, \{3\}, \emptyset)

is a finite simplicial complex, while the following is not:

([3], \{0\}, \ldots, \{3\}, \emptyset, \{1, 2, 3\})

Note: You can take the simplicial closure of the second example to form a simplicial complex:

\{\{0\}, \ldots, \{3\}, \emptyset, \{1, 2, 3\}\} \cup \{\{2, 3\}, \{1, 3\}, \{2, 3\}\}

The standard $n$-simplex is:

$$\Delta_n = (V = [n] = \{0, 1, 2, \ldots, n\}, C = 2^{[n]})$$

For example,

$$\Delta_1 = \{\emptyset, \{0, 1\}, \{0\}, \{1\}, \{0, 1\}$$