The portfolio for the course primarily consists of a project and related items which develops a topic related to graphs embedded on surfaces. An oral presentation of your project will be made during the last weeks of class. A written report will be submitted by the Final Exam date.

0.1 Calendar

- Proposal & First Review: Tuesday, 20 October Submit one written review & a description of your selected presentation topic with a brief summary and initial list of references.
- Written Portfolio: Due at Final Exam, Wed., 9 December at noon.

0.2 Additional Portfolio Item Choices

- Worked Exercises or Examples
- Revised & Expanded Lecture Notes for a single topic
- Algorithmic Development of a Topic (e.g. Mathematica/Maple notebook)
- **Required:** Research Article Reviews/Summaries: In the style of a Math Review write three one-page summaries of published articles related to graphs embedded on surfaces.

1 Sources

Two good sources for topics and articles to review are the bibliography in Lando/Zvonkin [LZ] and in the books by Leila Schneps. Searching for keywords or following the links in MathSciNet is also another good way to find articles of interest and your topic of choice.

1.1 Permutation Group Theory

1.2 Applications to Knot Theory

- Turaev genus applications:
  - Characterization of alternations
  - Bound on width of Khovanov homology
  - Bound on width of Heegaard-Floer homology
- Chmutov papers and preprints on polynomials of signed ribbon graphs.
- Non-orientable quasi-tree expansion (recent: see me if interested for preprints)
- Self-dual maps (Servatius references in [LZ].), p. 121.
- Braid group invariant of genus zero dessins in *Galois invariants of dessins*, by Jordan Ellenberg.

1.3 Combinatorial Polynomial Invariants

- Sergi Chmutov papers and preprints on polynomials of signed ribbon graphs: see arXiv.
- Four variable polynomial in arXiv:0903.5312: Graphs, links, and duality on surfaces by Vyacheslav Krushkal

1.4 Enumeration of Dessins

- One-faced maps: Section 3.1, p. 155
- Counting Trees, Section 1.5.2, p. 45
- Inversive Enumeration Section 2.2.3.4, p. 99
- Frobenius Formula and Examples: Section 1.1.1.12, p. 10-2 and Appendix A.1.5. Requires linear algebra traces of group representations.

1.5 Coverings

- Ritt’s Theorem on decomposable extensions, Section 1.7.2, p. 65
- Symmetric and Regular constellations Section 1.7.3, p.68.
1.6 Polytopes and Belyi Maps
- Klein’s Theorem, Examples, Section 2.3.5
- Archimedian Solids: Example 2.3.6 & Magot-Zvonkin paper [207]

1.7 Belyi Function Properties
- DSZ degree bounds, Section 2.5.1, p. 126.
- Jacobi Polynomial Examples, Section 2.5.2
- Fermat curves and graph genus of complete tri-partite graph, $K_{n,n,n}$, Section 2.5.3
- Polynomial Pell’s Equation, Section 2.5.6

1.8 Algorithm Implementation
- Implement an algorithm for computing Belyi functions. (Lando/Zvonkin p.86-7)
- Implement an algorithm for drawing dessins. (Lando/Zvonkin p.89)